PRACTICE MCQs

CLASS 12 MATHS (TERM - I)

APPLICATION OF DERIVATIVES

BY

learn-o-hub
learning simplified

Question 1:

The intervals in which the function $f(x) = x^2 - 4x + 6$ is strictly increasing, is

(a) $(-\infty, -2) \cup (2, \infty)$

(b) $(2, \infty)$

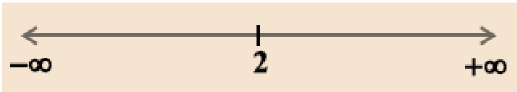
(c) $(-2, \infty)$

(d) $(-\infty, 2] \cup (2, \infty)$

Answer: (b) (2, ∞)

Given, $f(x) = x^2 - 4x + 6$

And $f'(x) = 2x - 4$



Therefore, $f'(x) = 0$ gives $x = 2$.

Now the point $x = 2$ divides the real line into two disjoint intervals $(-\infty, 2)$ and $(2, \infty)$ as shown in the figure.

In the interval $(-\infty, 2)$, $f'(x) = 2x - 4 < 0$

Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, $f'(x) > 0$

So, the function f is strictly increasing in $(2, \infty)$.

Question 2:

The functions cos 2x is strictly decreasing on

(a) $(0, \pi/3)$

(b) $(0, \pi/4)$

(c) $(0, \pi/2)$

(d) $(0, \pi)$

Answer: (c) (0, π/2)

Let f(x) = cos 2x

So, f'(x) = -2 sin 2x

Now, $0 < x < \pi/2$

=> $0 < 2x < \pi$

=> sin 2x > 0

=> -2 sin 2x < 0

So, f(x) = cos 2x is strictly decreasing in interval (0, π/2).

Question 3:

On which of the following intervals is the function f given by

$f(x) = x^{100} + \sin x - 1$ strictly decreasing?

(a) (0, 1)

(b) (π/2, π)

(c) (0, π/2)

(d) None of these

Answer: (d) None of these

Given, $f(x) = x^{100} + \sin x - 1$

So, $f'(x) = 100x^{99} + \cos x$

In interval (0, 1) cos x > 0 and $100x^{99} > 0$

Thus, function f is strictly increasing in interval (0, 1).

In interval (π/2, π), cos x < 0 and $100x^{99} > 0$

Also, $100x^{99} > \cos x$

So, $f'(x) > 0$ in $(\pi/2, \pi)$

Thus, function f is strictly increasing in interval $(\pi/2, \pi)$.

In interval $(0, \pi/2)$, $\cos x > 0$ and $100x^{99} > 0$

So, $100x^{99} + \cos x > 0$

=> $f'(x) > 0$ in $(0, \pi/2)$

Thus, function f is strictly increasing in interval $(0, \pi/2)$.

Hence, function f is strictly decreasing in none of the intervals.

Question 4:

The slope of the tangent to the curve $y = x^3 – x + 1$ at the point whose x-coordinate is 2, is

(a) 7

(b) 9

(c) 10

(d) 11

Answer: (d) 11

The given curve is $y = x^3 – x + 1$

Now, $dy/dx = 3x^2 – 1$

The slope of the tangent to a curve at (x_0, y_0) is $dy/dx]_{(x0,\ y0)}$

It is given that $x_0 = 2$

Then, the slope of the tangent to the given curve at $x = 2$ is given by,

$dy/dx]_{x = 2} = 3 * 2^2 – 1 = 12 – 1 = 11$

Question 5:

The point at which the tangent to the curve $y = \sqrt{(4x - 3)} - 1$ has its slope 2/3, is

(a) (1, 3)

(b) (2, 4)

(c) (3, 2)

(d) (1, 4)

Answer: (c) (3, 2)

Given, $y = \sqrt{(4x - 3)} - 1$

Slope of the tangent of the curve is given as

$dy/dx = (1/2) * (4x - 3)^{-1/2} * 4 = 2/\sqrt{(4x - 3)}$

Again, given slope = 2/3

=> $2/\sqrt{(4x - 3)} = 2/3$

=> $1/\sqrt{(4x - 3)} = 1/3$

=> $\sqrt{(4x - 3)} = 3$

=> $4x - 3 = 9$ [Squaring on both sides]

=> $4x = 12$

=> $x = 3$

Now, $y = \sqrt{(4 * 3 - 3)} - 1$

=> $y = \sqrt{(12 - 3)} - 1$

=> $y = \sqrt{9} - 1$

=> $y = 3 - 1$

=> $y = 2$

So, the required point is (3, 2).

Question 6:

The slope of the normal to the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ at $\theta = \pi/4$, is

(a) 0

(b) 1

(c) $1/\sqrt{3}$

(d) $\sqrt{3}$

Answer: (b) 1

Given, $x = a \cos^3 \theta$, $y = a \sin^3 \theta$

Now, $dx/d\theta = 3a \cos^2 \theta * (-\sin \theta) = -3a \cos^2 \theta * \sin \theta$

And $dy/d\theta = 3a \sin^2 \theta * \cos \theta$

So, $dy/dx = (dy/d\theta)/(dy/d\theta)$

$= (3a \sin^2 \theta * \cos \theta)/(-3a \cos^2 \theta * \sin \theta)$

$= - \sin \theta /\cos \theta$

$= - \tan \theta$

Therefore, the slope of the tangent at $\theta = \pi/4$ is given by,

$dy/dx]_{\theta = \pi/4} = -\tan \pi/4 = -1$

Now, the slope of the normal at $\theta = \pi/4 = -1/[dy/dx]_{\theta = \pi/4}$

$= -1/(-1)$

$= 1$

Question 7:

The equation of normal to the curve $x^{3/2} + y^{3/2} = 1$ at (1, 1), is

(a) $x – y = 0$

(b) $x + y = 0$

(c) x – y + 1 = 0

(d) x – y – 1 = 0

Answer: (a) x – y = 0

Given, $x^{3/2} + y^{3/2} = 1$

Differentiate w.r.t. x, we get

$(3/2)x^{1/2} + (3/2)y^{1/2} * (dy/dx) = 1$

=> $dy/dx = -(y/x)^{1/2}$

=> $dy/dx|_{(1,\ 1)} = -(1/1)^{1/2}$

=> $dy/dx|_{(1,\ 1)} = -1$

Now, equation of normal at (1, 1) is

y – 1 = {-1/(dy/dx)} * (x - 1)

=> y – 1 = {-1/(-1)} * (x - 1)

=> y – 1 = x – 1

=> x – y = 0

Question 8:

The point at which the normal to the curve $y = x + 1/x$, x > 0 is perpendicular to the line 3x – 4y – 7 = 0 is:

(a) (2, 5/2)

(b) (±2, 5/2)

(c) (-1/2, 5/2)

(d) (1/2, 5/2)

Answer: (a) (2, 5/2)

Given, $y = x + 1/x$

=> $dy/dx = 1 – 1/x^2$

Now, slope of the normal = $-1/(dy/dx) = -1/(1 – 1/x^2)$

Given, normal is perpendicular to $3x – 4y = 7$

=> Slope of normal * slope of line = -1

=> $-1/(1 – 1/x^2) * (3/4) = -1$

=> $1/(1 – 1/x^2) * (3/4) = 1$

=> $1 – 1/x^2 = 3/4$

=> $1 – 3/4 = 1/x^2$

=> $1/4 = 1/x^2$

=> $x^2 = 4$

=> $x = \pm 2$

Since, $x > 0$

So, $x = 2$

Now, $y = 2 + 1/2 = 5/2$

Thus, the point at which the normal is perpendicular to the line = (2, 5/2).

Question 9:

The curves $x = y^2$ and $xy = k$ cut at right angles if

(a) $k^2 = 1$

(b) $k^2 = 1/2$

(c) $k^2 = 1/4$

(d) $k^2 = 1/8$

Answer: (d) $k^2 = 1/8$

The equations of the given curves are given as $x = y^2$ and $xy = k$

Putting $x = y^2$ in $xy = k$, we get:

$y^3 = k$

=> $y = k^{1/3}$

So, $x = k^{2/3}$

Thus, the point of intersection of the given curves is $(k^{2/3}, k^{1/3})$.

Differentiating $x = y^2$ with respect to x, we have:

$1 = 2y * dy/dx$

=> $dy/dx = 1/2y$

Therefore, the slope of the tangent to the curve $x = y^2$ at $(k^{2/3}, k^{1/3})$ is

=> $dy/dx = 1/2k^{1/3}$

On differentiating $xy = k$ with respect to x, we have:

$x * dy/dx + y = 0$

=> $dy/dx = -y/x$

So, the slope of the tangent to the curve $xy = k$ at $(k^{2/3}, k^{1/3})$ is

=> $dy/dx = - k^{2/3}/k^{1/3} = -1/ k^{1/3}$

We know that two curves intersect at right angles if the tangents to the curves at the point of intersection i.e., at $(k^{2/3}, k^{1/3})$ are perpendicular to each other.

This implies that we should have the product of the tangents as − 1.

Thus, the given two curves cut at right angles if the product of the slopes of their respective tangents at is -1.

i.e. $(1/2k^{1/3}) * (-1/k^{1/3}) = -1$

=> $2k^{2/3} = 1$

=> $(2k^{2/3})^3 = 1^3$

=> $8k^2 = 1$

=> $k^2 = 1/8$

Hence, the given two curves cut at right angel if $k^2 = 1/8$.

Question 10:

The points on the curve $x^2/9 + y^2/16 = 1$ at which the tangents are parallel to y axis are:

(a) (0, ±4)

(b) (±4, 0)

(c) (±3, 0)

(d) (0, ±3)

Answer: (c) (±3, 0)

Given, $x^2/9 + y^2/16 = 1$ …….1

=> $y^2/16 = 1 - x^2/9$

Differentiate w.r.t. x, we get

=> $2y/16 * dy/dx = -2x/9$

=> $y/8 * dy/dx = -2x/9$

=> $dy/dx = (-2x/9)/(y/8)$

=> $dy/dx = (-16/9) *(x/y)$

Since tangents are parallel to y-axis

So, angle between x-axis = 90 degree

Now, slope = tan 90 = ∞

=> $dy/dx = \infty$

=> $dy/dx = 1/0$

=> $(-16/9) *(x/y) = 1/0$

=> $9y = 0$

=> $y = 0$

From equation 1, we get

$x^2/9 + 0 = 1$

=> $x^2 = 9$

=> $x = \pm 3$

Hence, the required point is (±3, 0).

Question 11:

The maximum value of the function given by $f(x) = |x|, x \in [-2, 1]$, is

(a) 0

(b) 1

(c) 2

(d) R

Answer: (c) 2

Given, f(x) = |x|, x ∈ [-2, 1]

Now f(-2) = |-2| = 2

And f(1) = |1| = 1

So, the maximum value of f is 2.

Question 12:

The local minimum value of the function f given by f(x) = 3 + |x|, x ∈ R, is

(a) 1

(b) 2

(c) 3

(d) No local minimum value exists

Answer: (c) 3

The given function is not differentiable at x = 0. So, second derivative test fails.

Let us try first derivative test.

Note that 0 is a critical point of f.

Now to the left of 0, f(x) = 3 – x and so f′(x) = – 1 < 0.

Also to the right of 0, f(x) = 3 + x and so f′(x) = 1 > 0.

Therefore, by first derivative test, x = 0 is a point of local minima of f and local minimum value of f is f(0) = 3.

Question 13:

The absolute maximum values of a function f given by f(x) = $2x^3 – 15x^2 + 36x + 1$ on the interval [1, 5], is

(a) 24

(b) 29

(c) 56

(d) 63

Answer: (c) 56

We have, $f(x) = 2x^3 – 15x^2 + 36x + 1$

Now, $f'(x) = 6x^2 – 30x + 36 = 6(x – 3)(x – 2)$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$.

We shall now evaluate the value of f at these points and at the end points of the interval [1, 5], i.e., at x = 1, x = 2, x = 3 and at x = 5.

$f (1) = 2(1^3) – 15 (1^2) + 36 (1) + 1 = 24$

$f (2) = 2(2^3) – 15 (2^2) + 36 (2) + 1 = 29$

$f (3) = 2(3^3) – 15 (3^2) + 36 (3) + 1 = 28$

$f (5) = 2(5^3) – 15 (5^2) + 36 (5) + 1 = 56$

Thus, the absolute maximum value of f on [1, 5] is 56, occurring at x = 5.

Question 14:

The function $h(x) = x^3 + x^2 + x + 1$ has

(a) one maxima and one minima

(b) one maxima only

(c) one minima only

(d) no maxima or minima

Answer: (d) no maxima or minima

We have, $h(x) = x^3 + x^2 + x + 1$

So, $h'(x) = 3x^2 + 2x + 1$

Now, h(x) = 0

So, $3x^2 + 2x + 1 = 0$

=> $x = (-2 \pm 2\sqrt{2}i)/6 = (-1 \pm \sqrt{2}i)/3 \notin R$

Therefore, there does not exist $c \in R$ such that h'(c) = 0

Hence, function h does not have maxima or minima.

Question 15:

It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Then the value of a is

(a) 80

(b) 120

(c) 160

(d) 200

Answer: (b) 120

Let $f(x) = x^4 - 62x^2 + ax + 9$

So, $f'(x) = 4x^3 - 124x + a$

Given that function f attains its maximum value on the interval [0, 2] at x = 1.

So, f'(1) = 0

=> $4 * 1^3 - 124 * 1 + a = 0$

=> 4 − 124 + a = 0

=> a = 120

Hence, the value of a is 120.

Question 16:

The least value of the function $f(x) = 2cos\ x + x$ in the closed interval [0, $\pi/2$] is:

(a) 2

(b) $\pi/6 + \sqrt{3}$

(c) $\pi/2$

(d) The least value does not exist.

Answer: (c) $\pi/2$

Given, f(x) = 2 cos x + x

Now, f'(x) = -2 sin x + 1

Now, f'(x) = 0

=> -2 sin x + 1 = 0

=> sin x = 1/2

=> sin x = π/6

Since we are given closed interval [0, π/2].

Let us check value of f(x) at 0, π/6 and π/2.

f(0) = 2 cos 0 + 0 = 2 * 1 = 2

f(π/6) = 2 cos π/6 + π/6 = 2 * √3/2 + π/6 = √3 + π/6

f(π/2) = 2 cos π/2 + π/2 = 2 * 0 + π/2 = π/2

So, the value of f(x) is lowest at x = π/2.

Question 17:

The maximum value of $[x(x-1)+1]^{1/3}$, $0 \leq x \leq 1$ is

(a) $(1/3)^{1/3}$

(b) 1/2

(c) 1

(d) 0

Answer: (c) 1

Let $f(x) = [x(x-1) + 1]^{1/3}$

So, $f'(x) = (2x - 1)/3[x(x-1) + 1]^{2/3}$

Now, $f'(x) = 0$

=> $(2x - 1)/3[x(x-1) + 1]^{2/3} = 0$

=> $2x - 1 = 0$

=> $x = 1/2$

Then, we evaluate the value of f at critical point x = 1/2 and at the end points of the interval [0, 1] {i.e., at x = 0 and x = 1}.

$f(0) = [0(0-1) + 1]^{1/3} = 1$

$f(1) = [1(1-1) + 1]^{1/3} = 1$

$f(1/2) = [(1/2)(1/2-1) + 1]^{1/3} = (3/4)^{1/3}$

So, the maximum value of f in the interval [0, 1] is 1.

Question 18:

The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is:

(a) Strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$

(b) Strictly decreasing in $(-2, 3)$

(c) Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$

(d) Strictly decreasing in $(-\infty, -2) \cup (3, \infty)$

Answer: (b) Strictly decreasing in (−2, 3)

Given, $f(x) = 2x^3 - 3x^2 - 36x + 7$

Now, $f'(x) = 6x^2 - 6x - 36$

=> $f'(x) = 6(x^2 - x - 6)$

=> $f'(x) = 6(x - 3)(x + 2)$

Now, put $f'(x) = 0$

=> $6(x - 3)(x + 2) = 0$

=> $(x - 3)(x + 2) = 0$

=> $x = -2, 3$

Value of x	Intervals	Sign of $f'(x) = 6(x+2)(x-3)$	Nature of function f
$x < -2$	$(-\infty, -2)$	$(+)(+) > 0$	f is strictly increasing
$-2 < x < 3$	$(-2, 3)$	$(+)(-) < 0$	f is strictly decreasing
$x > 3$	$(3, \infty)$	$(+)(+) > 0$	f is strictly increasing

From the above table,

f is strictly increasing in $(-\infty, -2) \cup (3, \infty)$ and strictly decreasing in $(-2, 3)$.

Question 19:

The value of b for which the function $f(x) = x + cosx + b$ is strictly decreasing over R is:

(a) $b < 1$

(b) No value of b exists

(c) $b \leq 1$

(d) $b \geq 1$

Answer: (b) No value of b exists

Given, f(x) = x + cos x + b

Now, f'(x) = 1 – sin x

Since f(x) is strictly decreasing over R

=> f'(x) < 0

=> 1 – sin x < 0

=> sin x > 1, which is not possible.

So, for no value of b, f(x) is strictly decreasing.

Question 20:

The area of a trapezium is defined by function f and given by

f(x) = $(10 + x)\sqrt{(100 - x^2)}$, then the area when it is maximised is:

(a) 75 cm^2

(b) $7\sqrt{3}$ cm^2

(c) $75\sqrt{3}$ cm^2

(d) 5 cm^2

Answer: (c) $75\sqrt{3}$ cm²

Given, f(x) = $(10 + x)\sqrt{(100 - x^2)}$

Let Z = $[f(x)]^2$

=> Z = $(10 + x)^2(100 - x^2)$

Now, Z' = $2(x + 10)(100 - x^2) - 2x(10 + x)^2$

=> Z' = $2(x + 10)[(100 - x^2 - x(10 + x)]$

=> Z' = $2(x + 10)[100 - x^2 - 10x - x^2]$

=> Z' = $2(x + 10)(-2x^2 - 10x + 100)$

=> Z' = $-4(x + 10)(x^2 + 5x + 50)$

Now, Z' = 0

=> $-4(x + 10)(x^2 + 5x + 50) = 0$

=> (x + 10)(x - 5)(x + 10) = 0

=> x = 5, -10

Since length cannot be negative,

So, x = 5

Now, maximum area of trapezium

$A = (10 + 5)\sqrt{(100 - 5^2)}$

$= 15\sqrt{(100 - 25)}$

$= 15\sqrt{(75)}$

$= 15\sqrt{(5 * 5 * 3)}$

$= 15 * 5\sqrt{3}$

$= 75\sqrt{3}\ cm^2$

Question 21:

The point(s) on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$ is/are:

(a) (-2, 19)

(b) (2, - 9)

(c) (±2, 19)

(d) (-2, 19) and (2, -9)

Answer: (b) (2, -9)

Given, $y = x^3 - 11x + 5$

Slope of tangent is dy/dx.

So, $dy/dx = 3x^2 - 11$ 1

Again given tangent is $y = x - 12$

Slope of tangent = 1 2

From equation 1 and 2, we get

$3x^2 - 11 = 1$

$\Rightarrow 3x^2 = 12$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Now, when x = 2

$y = 2^3 - 11 * 2 + 5$

$= 8 - 22 + 5$

$= -9$

So, the point is (2, -9).

And when x = -2

$y = (-2)^3 - 11 * (-2) + 5$

$= -8 + 22 + 5$

$= 19$

So, the point is (-2, 19).

But (-2, 19) does not satisfy the line y = x - 11

So, only point is (2, -9).

Question 22:

For which value of m is the line y = mx + 1 a tangent to the curve $y^2 = 4x$?

(a) 1/2

(b) 1

(c) 2

(d) 3

Answer: (b) 1

Let (h, k) be the point at which tangent is taken.

Since (h, k) lies on the line

So, k = mh + 1 1

Again, (h, k) lies on the curve

So, $k^2 = 4h$

=> $h = k^2/4$

Put value of h in equation 1, we get

$k = m(k^2/4) + 1$

=> $4k = mk^2 + 4$

=> $mk^2 - 4k + 4 = 0$

Since tangent touches the curve only at one point.

So, the above quadratic equation has only one root.

Now, discriminant of quadratic equation = 0

=> $(-4)^2 – 4 * m * 4 = 0$

=> 16 – 16m = 0

=> 16m = 16

=> m = 1

Question 23:

The maximum value of $[x(x - 1) + 1]^{1/3}$, $0 \le x \le 1$ is:

(a) 0

(b) 1/2

(c) 1

(d) $\sqrt[3]{(1/3)}$

Answer: (c) 1

Let $f(x) = [x(x - 1) + 1]^{1/3}$

=> $f(x) = [x^2 - x + 1]^{1/3}$

Now, $f'(x) = (1/3) * [x^2 - x + 1]^{1/3-1} * (2x - 1)$

=> $f'(x) = (1/3) * (x^2 - x + 1)^{-2/3} * (2x - 1)$

=> $f'(x) = (2x - 1)/\{3(x^2 - x + 1)^{-2/3}\}$

Now, put $f'(x) = 0$

=> $(2x - 1)/\{3(x^2 - x + 1)^{-2/3}\} = 0$

=> $2x - 1 = 0$

=> $x = 1/2$

Since $0 \le x \le 1$

Hence, the critical points are: x = 0, 1/2 and 1

Value of x	Value of $f(x) = [x(x-1)+1]^{\frac{1}{3}}$
At $x = 0$	$f(0) = [0(0-1)+1]^{\frac{1}{3}} = (1)^{\frac{1}{3}} = 1$
At $x = \frac{1}{2}$	$f\left(\frac{1}{2}\right) = \left[\frac{1}{2}\left(-\frac{1}{2}\right)+1\right]^{\frac{1}{3}} = \left[\frac{-1}{4}+1\right]^{\frac{1}{3}} = \left[\frac{3}{4}\right]^{\frac{1}{3}}$
At $x = 1$	$f(1) = [1(1-1)+1]^{\frac{1}{3}} = [0+1]^{\frac{1}{3}} = 1$

Hence, the maximum value is 1 when x = 0, 1

Case study based questions

Question 24:

$P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.

(i). What will be the production when the profit is maximum?

(a) 37500

(b) 12.5

(c) -12.5

(d) -37500

(ii). What will be the maximum profit?

(a) Rs 38,28,125

(b) Rs 38281.25

(c) Rs 39,000

(d) None

(iii). In which interval the profit is strictly increasing?

(a) (12.5, ∞)

(b) for all real numbers

(c) for all positive real numbers

(d) (0, 12.5)

(iv). When the production is 2 units what will be the profit of the company?

(a) 37500

(b) 37,730

(c) 37,770

(d) None

(v). What will be production of the company when the profit is Rs 38250?

(a) 15

(b) 30

(c) 2

(d) data is not sufficient to find

Answers:

(i). (b) 12.5

Given, $P(x) = -5x^2 + 125x + 37500$

Differentiate w.r.t. x, we get

$P'(x) = -10x + 125$ ……..1

Put $P'(x) = 0$, we get

$-10x + 125 = 0$

=> $-10x = -125$

=> $x = 125/10$

=> $x = 12.5$

Again differentiate equation 1 w.r.t. x, we get

$P''(x) = -10 < 0$

Since $P''(x) <$ at $x = 12.5$

So, $x = 12.5$ is point of maxima.

Thus, P(x) is maximum when $x = 12.5$

(ii). (b) Rs 38281.25

We know that P(x) is maximum when $x = 12.5$

Put $x = 12.5$ in P(x), we get

$P(x) = -5(12.5)^2 + 125 * 12.5 + 37500$

$= -5 * 156.25 + 1562.5 + 37500$

$= -781.25 + 1562.5 + 37500$

= Rs 38281.25

(iii). (d) (0, 12.5)

Profit is strictly increasing where

$P'(x) > 0$

$\Rightarrow -10x + 125 > 0$

$\Rightarrow 125 > 10x$

$\Rightarrow x < 125/10$

$\Rightarrow x < 12.5$

So, the profit is strictly increasing for $x \in (0, 12.5)$

(iv). (b) 37730

Put x = 2 in P(x), we get

$P(x) = -5(2)^2 + 125 * 2 + 37500$

$= -5 * 4 + 250 + 37500$

$= -20 + 250 + 37500$

$= 37730$

(v). (a) 15

Put P(x) = 38250, we get

$38250 = -5x^2 + 125x + 37500$

$\Rightarrow 5x^2 - 125x - 37500 + 38250 = 0$

$\Rightarrow 5x^2 - 125x + 750 = 0$

$\Rightarrow x^2 - 25x + 150 = 0$ [divide by 5]

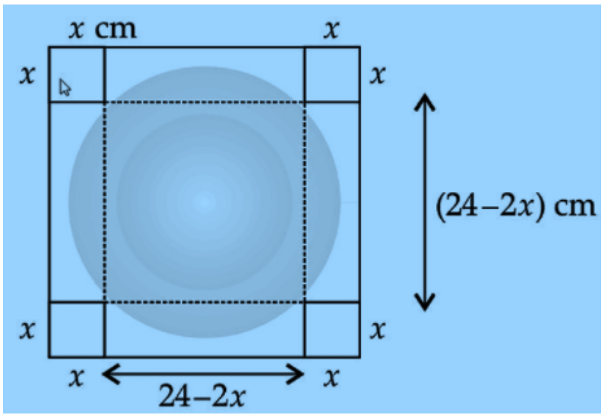
=> (x - 10)(x - 15) = 0

=> x = 10, 15

From the given options, we get x = 15

Question 25:

An open box is to be made out of a piece of cardboard measuring 24 cm * 24 cm by cutting of equal squares from the corners and turning up the sides.



Answer the following questions using the above information.

(i). The volume of the open box is

(a) $x^3 – 46x^2 + 568x$

(b) $2x^3 – 96x^2 + 226x$

(c) $4x^3 – 96x^2 + 576x$

(d) $6x^3 – 66x^2 + 676x$

(ii). If V is the volume of open box the dV/dx is

(a) $x^2 – 92x + 576$

(b) $9x^2 – 19x + 576$

(c) $12x^2 – 192x + 576$

(d) $16x^2 – 192x + 1324$

(iii). The value of d^2V/dx^2 is

(a) $12x + 88$

(b) $24x - 192$

(c) $36x + 288$

(d) $18x - 188$

(iv). The value of x other than 12 when volume is maximum, is

(a) 2

(b) 4

(c) 6

(d) No such value exists

(v). Volume is maximum at what height of the open box?

(a) 2

(b) 3

(c) 1

(d) 4

Answer:

(i). (c) $4x^3 – 96x^2 + 576x$

From the figure,

Height of the box = x cm

Length of the box = 24 – 2x cm

Breadth of the box = 24 - 2x cm

Now, volume of the box V = Length * breadth * height

= (24 – 2x) * (24 - 24) * x

$= x(24 – 2x)^2$

$= x(576 + 4x^2 – 96x)$

$= 576x + 4x^3 – 96x^2$

$= 4x^3 – 96x^2 + 576$

(ii). (c) $12x^2 – 192x + 576$

We have $V = 4x^3 – 96x^2 + 576x$

Now, $dV/dx = 4 * 3x^2 – 96 * 2x + 576 * 1$

=> $dV/dx = 12x^2 – 192x + 576$

(iii). (b) 24x - 192

We have $dV/dx = 12x^2 – 192x + 576$

Now, $d^2V/dx^2 = 12 * 2x – 192 * 1$

Now, $d^2V/dx^2 = 24x – 192$

(iv). (b) 4

We have $dV/dx = 12x^2 – 192x + 576$

For maxima and minima, $dV/dx = 0$

=> $12x^2 – 192x + 576 = 0$

=> $12(x^2 – 16x + 48) = 0$

=> $x^2 – 16x + 48 = 0$

=> $(x - 12)(x - 4) = 0$

=> $x = 4, 12$

Given, we have to take value of x other than 12

So, $x = 4$

Now, $d^2V/dx^2 = 24x – 192$

=> $d^2V/dx^2|_{x = 4} = 24 * 4 – 192$

=> $d^2V/dx^2|_{x = 4} = 96 – 192$

=> $d^2V/dx^2|_{x = 4} = -96 < 0$

So, x = 4 is point of maxima.

Thus, V(x) is maximum when x = 4.

(v). (d) 4

From (iv), volume is maximum at x = 4

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