PRACTICE MCQs

CLASS 12 MATHS (TERM - I)

CONTINUITY AND DIFFERENTIABILITY

BY

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learning simplified

Question 1:

Suppose f(x) and g(x) are two real functions continuous at a real number c.

Then which one is not correct?

(a) f(x) ± g(x) is continuous at x = c.

(b) f(x) * g(x) is continuous at x = c.

(c) f(x) /g(x) is continuous at x = c. (provided g (c) ≠ 0)

(d) None of these

Answer: (d) None of these

Given, f(x) and g(x) are two real functions continuous at a real number c.

Now,

-> f(x) + g(x) is continuous at x = c.

-> f(x) - g(x) is continuous at x = c.

-> f(x) * g(x) is continuous at x = c.

-> f(x) /g(x) is continuous at x = c. (provided g (c) ≠ 0)

Question 2:

The relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$$

is continuous at x = 3, is

(a) a = 3b + 2

(b) 2a = 3b + 2

(c) 3a = 3b + 2

(d) 4a = 3b + 2

Answer: (c) 3a = 3b + 2

The given function f is

$$f(x) = \begin{cases} ax + 1, \text{ if } x \leq 3 \\ bx + 3, \text{ if } x > 3 \end{cases}$$

If f is continuous at x = 3, then

$\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = 3a + 1$ ……….1

Also,

$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} (ax + 1) = 3a + 1$

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} (bx + 3) = 3b + 3$

Therefore, from equation 1, we get

$3a + 1 = 3b + 3 = 3a + 1$

=> $3a + 1 = 3b + 3$

=> $3a = 3b + 2$

Question 3:

For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x), \text{ if } x \leq 0 \\ 4x + 1, \text{ if } x > 0 \end{cases}$$

is continuous at x = 0?

(a) 1

(b) -1

(c) 2

(d) there is no value of λ

Answer: (d) there is no value of λ

The given function f is

$$f(x) = \begin{cases} \lambda(x^2 - 2x), \text{ if } x \leq 0 \\ 4x + 1, \text{ if } x > 0 \end{cases}$$

If f is continuous at x = 0, then

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

=> $\lim_{x\to 0^-} \{\lambda(x^2 - 2x)\} = \lim_{x\to 0^+} (4x + 1) = \lambda(0^2 - 2 * 0)$

=> $\lambda * 0 = 4 * 0 + 1$

=> 0 = 1, this is not possible.

Therefore, there is no value of λ for which f is continuous at x = 0.

Question 4:

The values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$

is a continuous function.

(a) a = 2, b = 5

(b) a = 2, b = 1

(c) a = 1, b = 2

(d) a = 5, b = 2

Answer: (b) a = 2, b = 1

The given function f is

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at x = 2 and x = 10

Since f is continuous at x = 2, we get

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

=> $\lim_{x\to 2^-} (5) = \lim_{x\to 2^+} (ax + b) = 5$

=> 5 = 2a + b = 5

=> 2a + b = 5 ………….1

Since f is continuous at x = 10, we get

$\lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$

=> $\lim_{x\to 10^-} (ax + b) = \lim_{x\to 10^+} (21) = 21$

=> 10a + b = 21 = 21

=> 10a + b = 21 ………….2

On subtracting equation 1 from equation 2, we get

8a = 16

=> a = 2

By putting a = 2 in equation 1, we obtain

2 * 2 + b = 5

=> 4 + b = 5

=> b = 1

Question 5:

The values of k so that the function f is continuous at π/2 is

$$f(x) = \begin{cases} (k * \cos x)/(\pi - 2x), & \text{if } x \neq \pi/2 \\ 3, & \text{if } x = \pi/2 \end{cases}$$

(a) 2

(b) 3

(c) 6

(d) 9

Answer: (c) 6

The given function f is

$$f(x) = \begin{cases} (k * \cos x)/(\pi - 2x), & \text{if } x \neq \pi/2 \\ 3, & \text{if } x = \pi/2 \end{cases}$$

The given function f is continuous at $x = \pi/2$, if f is defined at and if the value of the f at $x = \pi/2$ equals the limit of f at $x = \pi/2$

It is evident that f is defined at $x = \pi/2$ and $f(\pi/2) = 3$

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} [(k * \cos x)/(\pi - 2x)]$

Put $x = \pi/2 + h$

Then $x \to \pi/2$

$\Rightarrow h \to 0$

So, $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} [(k * \cos x)/(\pi - 2x)]$

$= \lim_{h \to 0} [\{k * \cos(\pi/2 + h)\}/\{\pi - 2(\pi/2 + h)\}]$

$= k * \lim_{h \to 0} [(-\sin h)/(-2h)]$

$= (k/2) * \lim_{h \to 0} [\sin h/h]$

$= (k/2) * 1$

$= k/2$

So, $\lim_{x \to \pi/2} f(x) = f(\pi/2)$

$\Rightarrow k/2 = 3$

$\Rightarrow k = 6$

Question 6:

Which of the following is a correct statement?

(a) Every continuous function is differentiable.

(b) Every differentiable function is continuous.

(c) Every differentiable function is not continuous.

(d) Only (a) and (c) are correct

Answer: (b) Every differentiable function is continuous.

If a function f is differentiable at a point c, then it is also continuous at that point. So, every differentiable function is continuous.

Question 7:

The value of k (k < 0) for which the function f defined as

$$f(x) = \begin{cases} (1-\cos kx)/x\sin x & , x \neq 0 \\ 1/2 & , x = 0 \end{cases}$$

is continuous at $x = 0$ is:

(a) ±1

(b) −1

(c) ±1/2

(d) 1/2

Answer: (b) -1

Given, function is continuous at x = 0

=> $\lim_{x \to 0} f(x) = f(0)$
=> $\lim_{x \to 0} \{(1 - \cos kx)/(x * \sin x)\} = 1/2$

=> $\lim_{x \to 0} \{(2 * \sin^2 kx/2)/(x * \sin x)\} = 1/2$

=> $2 * \lim_{x\to 0} [\{(\sin^2 kx/2)/(kx/2)^2\} * \{(kx/2)^2/(x * \sin x)\}] = 1/2$

=> $2 * \lim_{x\to 0} [\{(\sin^2 kx/2)/(kx/2)^2\} * \{(k^2x/4)/(\sin x)\}] = 1/2$

=> $(2k^2/4) * \lim_{x\to 0} [\{(\sin^2 kx/2)/(kx/2)^2\} * 1/(\sin x /x)] = 1/2$

=> $(2k^2/4) * [\lim_{x\to 0} \{(\sin^2 kx/2)/(kx/2)^2\} * \lim_{x\to 0} (x/\sin x)] = 1/2$

=> $(k^2/2) * 1 * 1 = 1/2$

=> $k^2 = 1$

=> $k = \pm 1$

Given, $k < 0$

So, $k = -1$

Question 8:

The derivative of $\sin^{-1}(2x\sqrt{(1 - x^2)})$ w.r.t. $\sin^{-1} x$, $1/\sqrt{2} < x < 1$, is

(a) 2

(b) $\pi/2 - 2$

(c) $\pi/2$

(d) -2

Answer: (a) 2

Let $y = \sin^{-1} x$

=> $x = \sin y$

We have to calculate the derivative of $\sin^{-1}(2x\sqrt{(1 - x^2)})$ w.r.t. $\sin^{-1} x$

= derivative of $\sin^{-1}(2x\sqrt{(1 - x^2)})$ w.r.t. y

= $d\{\sin^{-1}(2x\sqrt{(1 - x^2)})\}/dy$

Put x = sin y, we get

$= d\{\sin^{-1}(2\sin y \sqrt{(1 - \sin^2 y)})\}/dy$

$= d\{\sin^{-1}(2\sin y \cos y)\}/dy$

$= d\{\sin^{-1}(\sin 2y)\}/dy$

= d(2y)/dy

= 2

Question 9:

The derivative of $\sin^2 y + \cos xy = \pi$ w.r.t. x, is

(a) (y * sin xy)/(sin 2y – x * sin xy)

(b) (y * sin xy)/(sin 2y – xy * sin xy)

(c) (y * sin xy)/(sin 2y + x * sin xy)

(d) (y * sin xy)/(sin 2y + xy * sin xy)

Answer: (b) (y * sin xy)/(sin 2y – x * sin xy)

Given, $\sin^2 y + \cos xy = \pi$

Differentiating w.r.t. x, we get

=> $d(\sin^2 y + \cos xy)/dx = d(\pi)/dx$

=> $d(\sin^2 y)/dx + d(\cos xy)/dx = 0$

=> 2 sin y * d(sin y)/dx – sin xy * d(xy)/dx = 0

=> 2 sin y * cos y * dy/dx – sin xy * [y * d(x)/dx + x * dy/dx] = 0

=> 2 sin y * cos y * dy/dx – sin xy * [y + x * dy/dx] = 0

=> 2 sin y * cos y * dy/dx – y * sin xy – x * sin xy * dy/dx = 0

=> 2 sin y * cos y * dy/dx – x * sin xy * dy/dx = y * sin xy

=> (2 sin y * cos y – x * sin xy) * dy/dx = y * sin xy

=> dy/dx = (y * sin xy)/(2 sin y * cos y – x * sin xy)

=> dy/dx = (y * sin xy)/(sin 2y – x * sin xy)

Question 10:

If $e^x + e^y = e^{x+y}$ then dy/dx is

(a) e^{y-x}

(b) e^{x+y}

(c) $-e^{y-x}$

(d) $2e^{x-y}$

Answer: (c) $-e^{y-x}$

Given, $e^x + e^y = e^{x+y}$

=> $e^x + e^y = e^x * e^y$

Divide by $e^x * e^y$ on both sides, we get

=> $e^x/(e^x * e^y) + e^y/(e^x * e^y) = (e^x * e^y)/(e^x * e^y)$

=> $1/e^y + 1/e^x = 1$

=> $e^{-y} + e^{-x} = 1$

Differentiate w.r.t x, we get

$d(e^{-y} + e^{-x})/dx = d(1)/dx$

=> $-e^{-y} * dy/dx - e^{-x} = 0$

=> $-e^{-y} * dy/dx = e^{-x}$

=> $dy/dx = -e^{-x}/e^{-y}$

=> $dy/dx = -e^{-x+y}$

=> $dy/dx = -e^{y-x}$

Question 11:

If $y = \sec^{-1}\{1/(2x^2 - 1)\}$, $0 < x < 1/\sqrt{2}$, then dy/dx is

(a) $1/\sqrt{(1 + x^2)}$

(b) $-2/\sqrt{(1 + x^2)}$

(c) $-2/\sqrt{(1 - x^2)}$

(d) $-1/\sqrt{(1 + x^2)}$

Answer: (c) $-2/\sqrt{(1 - x^2)}$

Given, $y = \sec^{-1}\{1/(2x^2 - 1)\}$

=> $\sec y = 1/(2x^2 - 1)$

=> $1/\cos y = 1/(2x^2 - 1)$

=> $\cos y = 2x^2 - 1$

=> $2x^2 = 1 + \cos y$

=> $2x^2 = 2 \cos^2 y/2$

=> $x^2 = \cos^2 y/2$

=> $x = \cos y/2$

Differentiating w.r.t. x, we get

$d(x)/dx = d(\cos y/2)$

=> $1 = -\sin y/2 * d(y/2)/dx$

=> $1 = -\sin y/2 * (1/2) * dy/dx$

=> $(1/2) * dy/dx = -1/(\sin y/2)$

=> $dy/dx = -2/(\sin y/2)$

=> $dy/dx = -2/\sqrt{(1 - \cos^2 y/2)}$

=> $dy/dx = -2/\sqrt{(1 - x^2)}$

Question 12:

If $y = 5 \cos x - 3 \sin x$, then d^2y/dx^2 is equal to:

(a) -y

(b) y

(c) 25y

(d) 9y

Answer: (a) -y

Given, $y = 5 \cos x - 3 \sin x$ 1

Differentiate w.r.t. x, we get

$dy/dx = -5 \sin x - 3 \cos x$ 2

Again, differentiate w.r.t. x, we get

$d^2y/dx^2 = -5 \cos x - 3 (-\sin x)$

$d^2y/dx^2 = -5 \cos x + 3 \sin x$

$d^2y/dx^2 = -(5 \cos x - 3 \sin x)$

$d^2y/dx^2 = -y$ [From equation 1]

Question 13:

Differentiation of $\sin(\tan^{-1} x\, e^{-x})$ w.r.t. x: is

(a) $\{e^{-x} * \cos(\tan^{-1} e^{-x})\}/(1 + e^{-2x})$

(b) $\{e^{-x} * \cos(\tan^{-1} e^{-x})\}/(1 - e^{-2x})$

(c) $\{-e^{-x} * \cos(\tan^{-1} e^{-x})\}/(1 + e^{-2x})$

(d) $\{-e^{-x} * \cos(\tan^{-1} e^{-x})\}/(1 - e^{-2x})$

Answer: (c) $\{-e^{-x} * \cos(\tan^{-1} e^{-x})\}/(1 + e^{-2x})$

Let $y = \sin(\tan^{-1} e^{-x})$

Differentiate w.r.t. x, we get

$dy/dx = d\{\sin(\tan^{-1} e^{-x})\}/dx$

By using the chain rule, we obtain

$dy/dx = \cos(\tan^{-1} e^{-x}) * d(\tan^{-1} e^{-x})/dx$

$= \cos(\tan^{-1} e^{-x}) * 1/\{1 + (e^{-x})^2\} * d(e^{-x})/dx$

$= \cos(\tan^{-1} e^{-x}) * 1/\{1 + (e^{-x})^2\} * (-e^{-x})$

$= \{-e^{-x} * \cos(\tan^{-1} e^{-x})\}/(1 + e^{-2x})$

Question 14:

If $y = \log(\cos e^x)$ then dy/dx is

(a) $\cos e^{x-1}$

(b) $e^x \cos e^x$

(c) $e^x \sin e^x$

(d) $-e^x \tan e^x$

Answer: (d) $-e^x \tan e^x$

Given, $y = \log(\cos e^x)$

Differentiate w.r.t. x, we get

$dy/dx = d[\log(\cos e^x)]/dx$

$= 1/(\cos e^x) * d(\cos e^x)/dx$

$= 1/(\cos e^x) * (-\sin e^x) * d(e^x)/dx$

$= -\tan e^x * e^x$

$= -e^x * \tan e^x$

Question 15:

The value of dy/dx, if x = a(θ + sin θ), y = a(1 – cos θ), is

(a) cot θ/2

(b) tan θ/2

(c) sec θ/2

(d) cos θ/2

Answer: (b) tan θ/2

Given, x = a(θ + sin θ)

Differentiate w.r.t. θ, we get

dx/dθ = a(1 + cos θ)

Again, y = a(1 - cos θ)

Differentiate w.r.t. θ, we get

dy/dθ = a sin θ

Now, dy/dx = (dy/dθ)/(dx/dθ)

= a sin θ /a(1 + cos θ)

= sin θ /(1 + cos θ)

$= (2 * \sin \theta/2 * \cos \theta/2)/(2 * \cos^2 \theta/2)$

= (sin θ/2)/(cos θ/2)

= tan θ/2

Question 16:

If $x = 2at^2$, $y = at^4$ then dy/dx is

(a) t

(b) 2t

(c) t^2

(d) $2t^2$

Answer: (c) t^2

The given equations are: $x = 2at^2$, $y = at^4$

Now, $dx/dt = d(2at^2)/dt$

=> $dx/dt = 2a * d(t^2)/dt$

=> $dx/dt = 2a * 2t$

=> $dx/dt = 4at$

and $dy/dt = d(at^4)/dt$

=> $dx/dt = a * d(t^4)/dt$

=> $dx/dt = a * 4t^3$

=> $dx/dt = 4at^3$

So, $dy/dx = (dy/dt)/(dx/dt)$

=> $dy/dx = 4at^3/4at$

=> $dy/dx = t^2$

Question 17:

The point(s) at which the function f given by

$$f(x) = \begin{cases} x/|x|, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

is continuous, is/are:

(a) $x \in R$

(b) $x = 0$

(c) $x \in R - \{0\}$

(d) $x = -1$ and 1

Answer: (a) $x \in R$

The given function f is

$$f(x) = \begin{cases} x/|x|, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

It is known that, if $x < 0$

$\Rightarrow |x| = -x$

Therefore, the given function can be rewritten as

$$f(x) = \begin{cases} x/|x| = x/(-x) = -1, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

$\Rightarrow f(x) = -1$ for all $x \in R$

Let c be any real number. Then,

$\lim_{x\to c} f(x) = \lim_{x\to c} f(-1) = -1$

Also, $f(c) = -1 = \lim_{x\to c} f(x)$

Therefore, the given function is a continuous function.

Hence, the given function has no point of discontinuity.

Question 18:

The point(s) at which the function f given by

$f(x) = \begin{cases} \sin x / x, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$

is continuous, is/are:

(a) $x \in R$

(b) $x = 1$

(c) $x \in R - \{0, 1\}$

(d) $x = -1$ and 1

Answer: (a) $x \in R$

The given function f is

$f(x) = \begin{cases} \sin x / x, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

If $c < 0$, then $f(c) = \sin c / c$

And $\lim_{x \to c} f(x) = \lim_{x \to c} (\sin x / x) = \sin c / c$

So, $\lim_{x \to c} f(x) = f(c)$

Therefore, f is continuous at all points x, such that $x < 0$

Case II:
If $c > 0$, then $f(c) = c + 1$

And $\lim_{x \to c} f(x) = \lim_{x \to c} (x + 1) = c + 1$

So, $\lim_{x \to c} f(x) = f(c)$

Therefore, f is continuous at all points x, such that $x > 0$

Case III:

If $c = 0$, then $f(c) = f(0) = 0 + 1 = 1$

The left hand limit of f at $x = 0$ is,

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (\sin x / x) = 1$

The right hand limit of f at $x = 0$ is,

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x + 1) = 1$

So, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

Therefore, f is continuous at $x = 0$

From the above observations, it can be concluded that f is continuous at all points of the real line. Thus, f has no point of discontinuity.

Question 19:

If $y = Ae^{mx} + Be^{nx}$, then $d^2y/dx^2 - (m + x) * dy/dx =$

(a) -my

(b) -ny

(c) -mny

(d) y

Answer: (c) -mny

Given, $y = Ae^{mx} + Be^{nx}$

Differentiate w.r.t. t, we get

$dy/dx = d[Ae^{mx} + Be^{nx}]/dx$

=> $dy/dx = d(Ae^{mx})/dx + d(Be^{nx})/dx$

=> $dy/dx = Ame^{mx} + Bne^{nx}$

Again, differentiate w.r.t. t, we get

$d^2y/dx^2 = d[Ame^{mx} + Bne^{nx}]/dx$

=> $d^2y/dx^2 = d(Ame^{mx})/dx + d(Bne^{nx})/dx$

=> $d^2y/dx^2 = Am^2e^{mx} + Bn^2e^{nx}$

Now, $d^2y/dx^2 – (m + n) * dy/dx + mny$

$= Am^2e^{mx} + Bn^2e^{nx} – (m + n) * (Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$

$= Am^2e^{mx} + Bn^2e^{nx} – (Am^2e^{mx} + Bmne^{nx} + Amne^{mx} + Bn^2e^{nx}) + Amne^{mx} + Bmne^{nx}$

$= Am^2e^{mx} + Bn^2e^{nx} – Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + Amne^{mx} + Bmne^{nx}$

$= 0$

=> $d^2y/dx^2 – (m + n) * dy/dx + mny = 0$

=> $d^2y/dx^2 – (m + n) * dy/dx = -mny$

Question 20:

If $y = \sin^{-1} x$, then $(1 – x^2)d^2y/dx^2 – x * dy/dx =$

(a) 0

(b) 1

(c) y

(d) 2y

Answer: (a) 0

Given, $y = \sin^{-1} x$

Differentiate w.r.t. x, we get

$dy/dx = 1/\sqrt{(1 - x^2)}$

$\Rightarrow dy/dx * \sqrt{(1 - x^2)} = 1$

Again, differentiate w.r.t. x, we get

$\Rightarrow d[dy/dx * \sqrt{(1 - x^2)}]/dx = d(1)/dx$

$\Rightarrow \sqrt{(1 - x^2)} * d^2y/dx^2 + (dy/dx) * d\{\sqrt{(1 - x^2)}\}/dx = 0$

$\Rightarrow \sqrt{(1 - x^2)} * d^2y/dx^2 + (dy/dx) * \{-2x/2\sqrt{(1 - x^2)}\} = 0$

$\Rightarrow \sqrt{(1 - x^2)} * d^2y/dx^2 - (dy/dx) * \{x/\sqrt{(1 - x^2)}\} = 0$

$\Rightarrow (1 - x^2) * d^2y/dx^2 - x * (dy/dx) = 0$

Question 21:

If $e^y(x + 1) = 1$, show that $d^2y/dx^2 =$

(a) dy/dx

(b) $(dy/dx)^2$

(c) $-dy/dx$

(d) $-(dy/dx)^2$

Answer: (b) $(dy/dx)^2$

The given relationship is $e^y(x + 1) = 1$

$\Rightarrow e^y = 1/1(x + 1)$

Taking logarithm on both the sides, we obtain

$\Rightarrow y = \log\{1/1(x + 1)\}$

Differentiating this relationship with respect to x, we get

$dy/dx = 1/\{1/(x + 1)\} * d\{1/(x + 1)\}/dx$

$\Rightarrow dy/dx = (x + 1) * (-1)/(x + 1)^2$

=> $dy/dx = -1/(x + 1)$

Again, differentiate w.r.t. t, we get

$d^2y/dx^2 = d[-1/(x + 1)]/dx$

=> $d^2y/dx^2 = -d[1/(x + 1)]/dx$

=> $d^2y/dx^2 = -[(-1)/ (x + 1)^2]$

=> $d^2y/dx^2 = 1/ (x + 1)^2$

=> $d^2y/dx^2 = \{-1/(x + 1)\}^2$

=> $d^2y/dx^2 = (dy/dx)^2$

Question 22:

If $f(x) = \begin{cases} ax^2 + b, b \neq 0, & x \leq 1 \\ bx^2 + ax + c, & x > 1 \end{cases}$, then f(x) is continuous and differentiable at x = 1 if

(a) $c = 0, a = 2b$

(b) $c \in R, a = b$

(c) $a = b, c = 0$

(d) $a = b, c \neq 0$

Answer: (a) c = 0, a = 2b

Since f(x) is continuous at x = 1, therefore

$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$

=> $a + b = b + a + c$

=> $c = 0$

Again f(x) is differentiable at x = 1, therefore

(LHD at x = 1) = (RHD at x = 1)

=> $2a + 0 = 2b + a$

=> $a = 2b$

Hence, $a = 2b$, $c = 0$

Question 23:

If $x = a \sec \theta$, $y = b \tan \theta$ then d^2y/dx^2 at $\theta = \pi/6$ is

(a) $(-3\sqrt{3}b)/a^2$

(b) $(-2\sqrt{3}b)/a$

(c) $(-3\sqrt{3}b)/a$

(d) $-b/(3\sqrt{3}a^2)$

Answer: (a) $(-3\sqrt{3}b)/a^2$

Given, $x = a \sec \theta$, $y = b \tan \theta$

Now, $dy/dx = (dy/d\theta)/(dx/d\theta)$

$= (b * \sec^2 \theta)/(a * \sec \theta * \tan \theta)$

$= (b/a) * (\sec \theta / \tan \theta)$

$= (b/a) * \{(1/\cos \theta)/(\sin \theta / \cos \theta)\}$

$= (b/a) * \text{cosec}\, \theta$

Now, $d^2y/dx^2 = (b/a) * d(\text{cosec}\, \theta)/d\theta * (d\theta/dx)$

=> $d^2y/dx^2 = (b/a) * d(\text{cosec}\, \theta)/d\theta * 1/(dx/d\theta)$

=> $d^2y/dx^2 = (b/a) * (-\text{cosec}\, \theta * \cot \theta) * 1/(a * \sec \theta * \tan \theta)$

=> $d^2y/dx^2 = -(b/a^2) * (1/\sin \theta * \cot \theta) * \cos \theta * \cot \theta$

=> $d^2y/dx^2 = -(b/a^2) * \cot \theta * \cot \theta * \cot \theta$

=> $d^2y/dx^2 = -(b/a^2) * \cot^3 \theta$

=> $d^2y/dx^2 \,|_{\theta = \pi/6} = -(b/a^2) * \cot^3 \pi/6$

=> $d^2y/dx^2 \,|_{\theta = \pi/6} = -(b/a^2) * (\sqrt{3})^3$

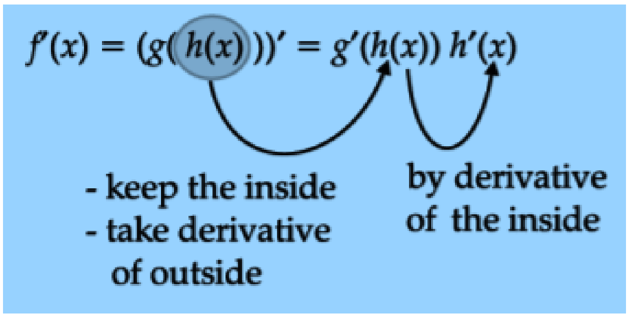
=> $d^2y/dx^2 \,|_{\theta = \pi/6} = -3\sqrt{3}b/a^2$

Case study based questions

Question 24:

Read the following text and answer the following questions.

Ms. Reema of city school is teaching chain rule to her students with the help of a flow-chart. The chain rule says that if h and g are functions and f(x) = g(h(x)), then



Let f(x) = sin x and g(x) = x^3

(i). fog(x) =

(a) sin x^3

(b) $\sin^3 x$

(c) sin 3x

(d) 3 sin x

(ii). gof(x) =

(a) sin x^3

(b) $\sin^3 x$

(c) sin 3x

(d) 3 sin x

(iii). $d(\sin x^3)/dx =$

(a) $\cos x^3$

(b) $-\cos x^3$

(c) $3x^2 * \sin x^3$

(d) $3x^2 * \cos x^3$

(iv). $d(\sin^3 x)/dx =$

(a) $\cos^3 x$

(b) $3 * \sin x * \cos x$

(c) $3 * \sin^2 x * \cos x$

(d) $-\cos^3 x$

(v). $d(\sin 2x)/dx$ at $x = \pi/2$, is

(a) 0

(b) -1

(c) -1/2

(d) -2

Answers:

(i). (a) $\sin x^3$

Given, $f(x) = \sin x$ and $g(x) = x^3$

Now, $fog(x) = f(g(x))$

$= f(x^3)$

$= \sin x^3$

(ii). (b) $\sin^3 x$

Given, $f(x) = \sin x$ and $g(x) = x^3$

Now, gof(x) = g(f(x))

= g(sin x)

= $\sin^3 x$

(iii). (d) $3x^2 * \cos x^3$

$d(\sin x^3)/dx = \cos x^3 * d(x^3)/dx$

$= \cos x^3 * 3x^2$

$= 3x^2 * \cos x^3$

(iv). (c) $3 * \sin^2 x * \cos x$

$d(\sin^3 x)/dx = 3 * \sin^2 x * d(\sin x)/dx$

$= 3 * \sin^2 x * \cos x$

(v). (d) -2

d(sin 2x)/dx = cos 2x * d(2x)/dx

= 2 cos 2x

Now, $d(\sin 2x)/dx|_{x = \pi/2} = 2 * \cos (2 * \pi/2)$

$= 2 * \cos \pi$

= 2 * (-1)

= -2

Question 25:

A potter made a mud vessel, where the shape of the pot is based on

f(x) = |x – 3| + |x - 2|, where f(x) represents the height of the pot.

(i). When x > 4, what will be the height in terms of x?

(a) x - 2

(b) x - 3

(c) 2x - 5

(d) 5 – 2x

(ii). Will the slope vary with x value?

(a) Yes

(b) No

(iii). What is dy/dx at x = 3

(a) 2

(b) -2

(c) Function is not differentiable

(d) 1

(iv). When the x values lies between (2, 3), then the function is

(a) 2x - 5

(b) 5 – 2x

(c) 1

(d) 5

(v). If the potter is trying to make a pot using the function f(x) = [x], will he get a pot or not? Why?

(a) Yes, because it is a continuous function

(b) Yes, because it is not continuous

(c) No , because it is a continuous function

(d) No , because it is not continuous

Answer:

Given, $f(x) = |x - 3| + |x - 2|$

$$= \begin{cases} (x - 3) + (x - 2), & x \geq 3 \\ -(x - 3) + (x - 2), & 2 < x < 3 \\ -(x - 3) - (x - 2), & x \leq 2 \end{cases}$$

$$= \begin{cases} 2x - 5, & x \geq 3 \\ -x + 3 + x - 2, & 2 < x < 3 \\ -x + 3 - x + 2, & x \leq 2 \end{cases}$$

$$= \begin{cases} 2x - 5, & x \geq 3 \\ 1, & 2 < x < 3 \\ -2x + 5, & x \leq 2 \end{cases}$$

(i). (c) 2x - 5

Given,

$$f(x) = \begin{cases} 2x - 5, & x \geq 3 \\ 1, & 2 < x < 3 \\ -2x + 5, & x \leq 2 \end{cases}$$

For $x > 4$, $f(x) = 2x - 5$

(ii). (a) Yes

Given,

$$f(x) = \begin{cases} 2x - 5, & x \geq 3 \\ 1, & 2 < x < 3 \\ -2x + 5, & x \leq 2 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x \geq 3 \\ 0, & 2 < x < 3 \\ -2, & x \leq 2 \end{cases}$$

Here, slope changes as value of x changes.

(iii). (c) Function is not differentiable

Given,

$$f(x) = \begin{cases} 2x - 5, & x \geq 3 \\ 1, & 2 < x < 3 \\ -2x + 5, & x \leq 2 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x \geq 3 \\ 0, & 2 < x < 3 \\ -2, & x \leq 2 \end{cases}$$

Since on left and right sides of $x = 3$, value of $f'(x)$ is different.

So, the function is not differentiable.

(iv). (c) 1

Given,

$$f(x) = \begin{cases} 2x - 5, & x \geq 3 \\ 1, & 2 < x < 3 \\ -2x + 5, & x \leq 2 \end{cases}$$

For $2 < x < 3$, $f(x) = 1$

(v). (d) No, because it is not continuous

Given, $f(x) = [x]$ = Greatest integer function

And Greatest integer function is not continuous.

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