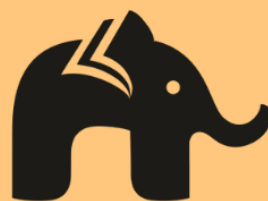


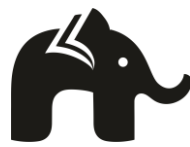


PRACTICE MCQS

CLASS 12 MATHS (TERM - I)
**INVERSE TRIGONOMETRIC
FUNCTIONS**

BY
learn-o-hub
learning simplified



**Question 1:**

Principal value of $\cot^{-1}(-1/\sqrt{3})$ is

- (a) $\pi/3$
- (b) $2\pi/3$
- (c) $\pi/6$
- (d) $\pi/4$

Answer: (b) $2\pi/3$

Let $\cot^{-1}(-1/\sqrt{3}) = y$, then

$$\cot y = -1/\sqrt{3} = -\cot(\pi/3) = \cot(\pi - \pi/3) = \cot(2\pi/3)$$

We know that the range of the principal value branch of \cot^{-1} is $(0, \pi)$

$$\text{and } \cot(2\pi/3) = -1/\sqrt{3}$$

Hence, the principal value of $\cot^{-1}(-1/\sqrt{3}) = 2\pi/3$.

Question 2:

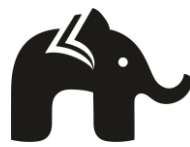
The principal value of $\sec^{-1}(2/\sqrt{3})$ is

- (a) $\pi/3$
- (b) $2\pi/3$
- (c) $\pi/6$
- (d) $\pi/4$

Answer: (c) $\pi/6$

Let $\sec^{-1}(2/\sqrt{3}) = y$, then

$$\sec y = 2/\sqrt{3} = \sec(\pi/6)$$



We know that the range of the principal value branch of \tan^{-1} is $[0, \pi] - \{\pi/2\}$

and $\sec(\pi/6) = 2/\sqrt{3}$

Hence, the principal value of $\sec^{-1}(2/\sqrt{3}) = \pi/6$.

Question 3:

$\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

- (a) π
- (b) $-\pi/3$
- (c) $\pi/3$
- (d) $2\pi/3$

Answer: (b) $-\pi/3$

Let $\tan^{-1}(\sqrt{3}) = y$, then

$$\tan y = \sqrt{3} = \tan \pi/3$$

We know that the range of the principal value branch of \tan^{-1} is $(-\pi/2, \pi/2)$

$$\text{So, } \tan^{-1}(\sqrt{3}) = \pi/3$$

Let $\sec^{-1}(-2) = y$, then

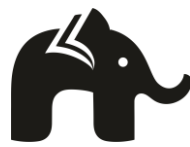
$$\sec y = -2 = -\sec \pi/3 = \sec(\pi - \pi/3) = \sec 2\pi/3$$

We know that the range of the principal value branch of \sec^{-1} is $[0, \pi] - \{\pi/2\}$

$$\text{So, } \sec^{-1}(-2) = 2\pi/3$$

$$\text{Now, } \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \pi/3 - 2\pi/3$$

$$= -\pi/3$$

**Question 4:**

$\sin(\pi/3 - \sin^{-1}(-1/2))$ is equal to

- (a) $1/2$
- (b) $1/3$
- (c) -1
- (d) 1

Answer: (d) 1

Given, $\sin(\pi/3 - \sin^{-1}(-1/2))$

We know that the range of the principal value branch of \sin^{-1} is $[-\pi/2, \pi/2]$

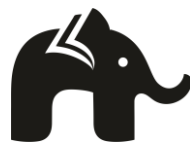
$$\begin{aligned}\text{So, } \sin(\pi/3 - \sin^{-1}(-1/2)) &= \sin[\pi/3 - \sin^{-1}(-\sin \pi/6)] \\ &= \sin[\pi/3 - \sin^{-1}(\sin(-\pi/6))] \\ &= \sin(\pi/3 + \pi/6) \\ &= \sin 3\pi/6 \\ &= \sin \pi/2 \\ &= 1\end{aligned}$$

Question 5:

If $\tan^{-1} x = y$, then

- (a) $-1 < y < 1$
- (b) $-\pi/2 \leq y \leq \pi/2$
- (c) $-\pi/2 < y < \pi/2$
- (d) $y \in \{-\pi/2, \pi/2\}$

Answer: (c) $-\pi/2 < y < \pi/2$



Given, $\tan^{-1} x = y$

We know that, $\tan x$ is not defined at $x = -\pi/2$ and $x = \pi/2$.

So, the range of $\tan^{-1} x$ excludes $-\pi/2$ and $\pi/2$.

Question 6:

The simplest form of the function $\tan^{-1}\{\sqrt{(1+x^2)} - 1\}/x$, $x \neq 0$, is

- (a) $\pi/2 - \sin^{-1} x$
- (b) $\pi/2 - \operatorname{cosec}^{-1} x$
- (c) $\pi/2 - \sec^{-1} x$
- (d) $\pi/2 - \cot^{-1} x$

Answer: (c) $\pi/2 - \sec^{-1} x$

Let $x = \operatorname{cosec} y$

Now, $\tan^{-1}\{1/\sqrt{(1+x^2)}\} = \tan^{-1}\{1/\sqrt{(\operatorname{cosec}^2 y - 1)}\}$

$$= \tan^{-1}[1/\cot y]$$

$$= \tan^{-1}[\tan y]$$

$$= y$$

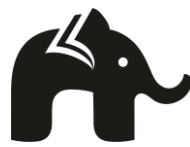
$$= \operatorname{cosec}^{-1} x$$

$$= \pi/2 - \sec^{-1} x$$

Question 7:

The value of $\tan^{-1}[2 \cos(2 \sin^{-1} 1/2)]$ is

- (a) $\pi/2$
- (b) $\pi/3$
- (c) $\pi/4$



(d) $\pi/6$

Answer: (c) $\pi/4$

$$\begin{aligned}\text{Given, } \tan^{-1}[2 \cos(2 \sin^{-1} 1/2)] &= \tan^{-1}[2 \cos(2 \sin^{-1} (\sin \pi/6))] \\ &= \tan^{-1}[2 \cos(2 * \pi/6)] \\ &= \tan^{-1}[2 \cos(\pi/3)] \\ &= \tan^{-1}[2 * 1/2] \\ &= \tan^{-1}[1] \\ &= \pi/4\end{aligned}$$

Question 8:

$\cos^{-1}(\cos 7\pi/6)$ is equal to

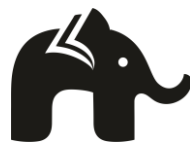
- (a) $7\pi/6$
- (b) $5\pi/6$
- (c) $\pi/3$
- (d) $\pi/6$

Answer: (c) $5\pi/6$

Given, $\cos^{-1}(\cos 7\pi/6)$

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1} x$.

So, $\cos^{-1}(\cos 7\pi/6) = \cos^{-1}(\cos \{2\pi - 5\pi/6\})$



$$= \cos^{-1}(\cos 5\pi/6)$$

$$= 5\pi/6 \in [0, \pi]$$

Hence, $\cos^{-1}(\cos 7\pi/6) = 5\pi/6$

Question 9:

If $\tan^{-1}(\cot \theta) = 2\theta$, then θ is equal to

- (a) $\pi/3$
- (b) $\pi/4$
- (c) $\pi/6$
- (d) $\pi/2$

Answer: (c) $\pi/6$

Given, $\tan^{-1}(\cot \theta) = 2\theta$

$$\Rightarrow \cot \theta = \tan 2\theta$$

$$\Rightarrow \tan(\pi/2 - \theta) = \tan 2\theta$$

$$\Rightarrow \pi/2 - \theta = 2\theta$$

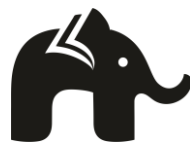
$$\Rightarrow \pi/2 = 3\theta$$

$$\Rightarrow \theta = \pi/6$$

Question 10:

Which of the following is the principal value branch of $\cos^{-1} x$

- (a) $[0, \pi]$
- (b) $(0, \pi) - \{\pi/2\}$
- (c) $[-\pi/2, \pi/2]$
- (d) $(0, \pi)$



Answer: (a) $[0, \pi]$

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1} x$.

Question 11:

The value of $\tan(\cos^{-1} x)$ is

- (a) $\sqrt{1 + x^2}/x$
- (b) $\sqrt{1 - x^2}/x$
- (c) $x/\sqrt{1 + x^2}$
- (d) $x/\sqrt{1 - x^2}$

Answer: (b) $\sqrt{1 - x^2}/x$

Let $\cos^{-1} x = \theta$

$\Rightarrow \cos \theta = x$

Now, $\tan \theta = \sqrt{\sec^2 \theta - 1}$

$$= \sqrt{1/x^2 - 1}$$

$$= \sqrt{(1 - x^2)/x^2}$$

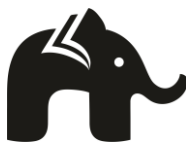
$$= \sqrt{1 - x^2}/x$$

$\Rightarrow \tan(\cos^{-1} x) = \sqrt{1 - x^2}/x$

Question 12:

If $\pi/2 \leq x \leq 3\pi/2$, then $\sin^{-1}(\sin x)$ is

- (a) x
- (b) $-x$
- (c) $\pi + x$
- (d) $\pi - x$



Answer: (d) $\pi - x$

Given, $\pi/2 \leq x \leq 3\pi/2$

$$\Rightarrow \pi/2 - \pi \leq x - \pi \leq 3\pi/2 - \pi$$

$$\Rightarrow -\pi/2 \leq x - \pi \leq \pi/2$$

$$\Rightarrow -\pi/2 \leq \pi - x \leq \pi/2$$

$$\Rightarrow \sin^{-1}(\sin x) = \pi - x$$

Question 13:

$\sin(\tan^{-1} x)$, where $|x| < 1$, is equal to

(a) $x/\sqrt{1 - x^2}$

(b) $1/\sqrt{1 - x^2}$

(c) $1/\sqrt{1 + x^2}$

(d) $x/\sqrt{1 + x^2}$

Answer: (d) $x/\sqrt{1 + x^2}$

Given, $\sin(\tan^{-1} x) = \sin(\sin^{-1}\{x/\sqrt{1 + x^2}\})$ [Since $\tan^{-1} a/b = \sin^{-1}\{a/\sqrt{a^2 + b^2}\}$]

$$= x/\sqrt{1 + x^2}$$

Question 14:

Simplest form of $\tan^{-1} \left[\frac{\sqrt{1 + \cos x} + \sqrt{1 - \cos x}}{\sqrt{1 + \cos x} - \sqrt{1 - \cos x}} \right]$,

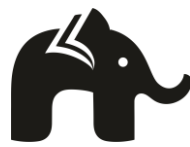
$\pi < x < 3\pi/2$, is

(a) $\pi/4 - x/2$

(b) $3\pi/2 - x/2$

(c) $-x/2$

(d) $\pi - x/2$



Answer: (a) $\pi/4 - x/2$

Given, $\tan^{-1} \left[\frac{\sqrt{1 + \cos x} + \sqrt{1 - \cos x}}{\sqrt{1 + \cos x} - \sqrt{1 - \cos x}} \right]$

We know that $\cos 2x = 2 * \cos^2 x - 1$

Replace x by $x/2$, we get

$$\cos x = 2 * \cos^2 x/2 - 1$$

$$\Rightarrow 1 + \cos x = 2 * \cos^2 x/2$$

$$\Rightarrow \sqrt{1 + \cos x} = \pm \sqrt{2 * \cos^2 x/2}$$

$$\Rightarrow \sqrt{1 + \cos x} = \pm \sqrt{2} * \cos x/2$$

Since x lies in 3rd quadrant, so $x/2$ lies in 2nd quadrant

So, $\sqrt{1 + \cos x} = -\sqrt{2} * \cos x/2$ [cos function is negative in 2nd quadrant]

Again, we know that $\cos 2x = 1 - 2 * \sin^2 x$

Replace x by $x/2$, we get

$$\cos x = 1 - 2 * \sin^2 x/2$$

$$\Rightarrow 1 - \cos x = 2 * \sin^2 x/2$$

$$\Rightarrow \sqrt{1 - \cos x} = \pm \sqrt{2 * \sin^2 x/2}$$

$$\Rightarrow \sqrt{1 - \cos x} = \pm \sqrt{2} * \sin x/2$$

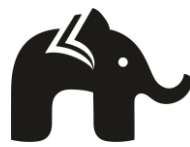
Since x lies in 3rd quadrant, so $x/2$ lies in 2nd quadrant

So, $\sqrt{1 - \cos x} = \sqrt{2} * \sin x/2$ [sin function is positive in 2nd quadrant]

Now, $\tan^{-1} \left[\frac{\sqrt{1 + \cos x} + \sqrt{1 - \cos x}}{\sqrt{1 + \cos x} - \sqrt{1 - \cos x}} \right]$

$$= \tan^{-1} \left[\frac{-\sqrt{2} * \cos x/2 + \sqrt{2} * \sin x/2}{-\sqrt{2} * \cos x/2 - \sqrt{2} * \sin x/2} \right]$$

$$= \tan^{-1} \left[\frac{(\cos x/2 - \sin x/2)}{(\cos x/2 + \sin x/2)} \right]$$



Divide by $\cos x/2$ in numerator and denominator

$$= \tan^{-1} \left[\frac{\{\cos x/2 - \sin x/2\}/\cos x/2}{\{\cos x/2 + \sin x/2\}/\cos x/2} \right]$$

$$= \tan^{-1} \left[\frac{1 - \tan x/2}{1 + \tan x/2} \right]$$

$$= \tan^{-1} \left[\frac{\{\tan \pi/4 - \tan x/2\}}{1 + \tan \pi/4 * \tan x/2} \right] \quad [\text{Since } \tan \pi/4 = 1]$$

$$= \tan^{-1} \{ \tan (\pi/4 - x/2) \}$$

$$= \pi/4 - x/2$$

Question 15:

$\sin^{-1}(1 - x) - 2\sin^{-1} x = \pi/2$, then x is equal to

(a) $0, 1/2$

(b) $1, 1/2$

(c) 0

(d) $1/2$

Answer: (c) 0

Given, $\sin^{-1}(1 - x) - 2\sin^{-1} x = \pi/2$

Let $x = \sin y$

So, $\sin^{-1}(1 - \sin y) - 2y = \pi/2$

$$\Rightarrow \sin^{-1}(1 - \sin y) = \pi/2 + 2y$$

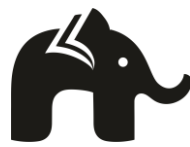
$$\Rightarrow 1 - \sin y = \sin(\pi/2 + 2y)$$

$$\Rightarrow 1 - \sin y = \cos 2y$$

$$\Rightarrow 1 - \sin y = 1 - 2\sin^2 y$$

$$\Rightarrow 2 * \sin^2 y - \sin y = 0$$

$$\Rightarrow 2x^2 - x = 0 \quad [\text{Since } x = \sin y]$$



$$\Rightarrow x(2x - 1) = 0$$

$$\Rightarrow x = 0, 1/2$$

But $x = 1/2$ does not satisfy the given equation.

So, $x = 0$ is the solution of the given equation

Question 16:

The value of $\cos^{-1}(\cos 13\pi/6)$ is

(a) $\pi/2$

(b) $\pi/3$

(c) $\pi/4$

(d) $\pi/6$

Answer: (d) $\pi/6$

Given, $\cos^{-1}(\cos 13\pi/6)$

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1} x$.

$$\text{So, } \cos^{-1}(\cos 13\pi/6) = \cos^{-1}(\cos \{2\pi + \pi/6\}) = \cos^{-1}(\cos \pi/6) = \pi/6 \in [0, \pi]$$

$$\text{Hence, } \cos^{-1}(\cos 13\pi/6) = \pi/6$$

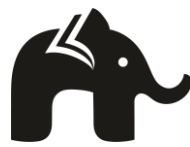
Question 17:

Simplest form of $\cot^{-1}\{1/\sqrt{x^2 - 1}\}$, $|x| > 1$, is

(a) $\operatorname{cosec}^{-1} x$

(b) $\tan^{-1} x$

(c) $\sec^{-1} x$



(d) $\cos^{-1} x$

Answer: (c) $\sec^{-1} x$

Let $x = \sec \theta$

So, $\sqrt{x^2 - 1} = \sqrt{\sec^2 \theta - 1} = \sqrt{\tan^2 \theta} = \tan \theta$

Now, $\cot^{-1}\{1/\sqrt{x^2 - 1}\} = \cot^{-1}(1/\tan \theta) = \cot^{-1}(\cot \theta) = \theta = \sec^{-1} x$

Question 18:

The function $\tan^{-1}[x/\sqrt{a^2 - x^2}]$, $|x| < a$ in the simplest form is

(a) $\sin^{-1} (x)$

(b) $\sin^{-1} (x/a)$

(c) $\sin^{-1} (2x/a)$

(d) $\sin^{-1} (x/2a)$

Answer: (b) $\sin^{-1} (x/a)$

Let $x = a \sin y$

Now, $\tan^{-1}[x/\sqrt{a^2 - x^2}] = \tan^{-1}[a \sin y/\sqrt{a^2 - a^2 \sin^2 y}]$

$$= \tan^{-1}[a \sin y/\{a\sqrt{1 - \sin^2 y}\}]$$

$$= \tan^{-1}[(a \sin y)/(a \cos y)]$$

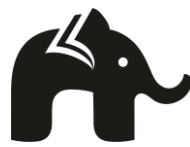
$$= \tan^{-1}(\tan y)$$

$$= y$$

$$= \sin^{-1} (x/a)$$

Question 19:

Find the value of $\sin^{-1}[\sin(-17\pi/8)]$



(a) $-\pi/2$

(b) $-\pi/4$

(c) $-\pi/6$

(d) $-\pi/8$

Answer: (d) $-\pi/8$

$$\begin{aligned} \sin^{-1}[\sin(-17\pi/8)] &= \sin^{-1}[-\sin(17\pi/8)] \\ &= \sin^{-1}[-\sin(2\pi + \pi/8)] \\ &= \sin^{-1}[-\sin \pi/8] \\ &= \sin^{-1}[\sin(-\pi/8)] \\ &= -\pi/8 \end{aligned}$$

Question 20:The principal value of $\tan^{-1} 1 + \cos^{-1} (-1/2)$, is

(a) $\pi/12$

(b) $5\pi/12$

(c) $11\pi/12$

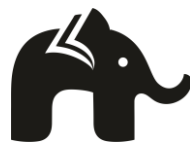
(d) None of these

Answer: (c) $11\pi/12$

Let $\tan^{-1} 1 = y$ and $\cos^{-1} (-1/2) = z$

$$\Rightarrow \tan y = 1 = \tan \pi/4 \text{ and } \cos z = -1/2 = -\cos \pi/3 = \cos(\pi - \pi/3) = \cos 2\pi/3$$

The range of principal value branch of \tan^{-1} and \cos^{-1} are $(-\pi/2, \pi/2)$ and $[0, \pi]$ respectively.



So, $\tan^{-1} 1 = \pi/4$ and $\cos^{-1} (-1/2) = 2\pi/3$

Therefore, $\tan^{-1} 1 + \cos^{-1} (-1/2) = \pi/4 + 2\pi/3 = 11\pi/12$

Question 21:

If $6 * \sin^{-1} (x^2 - 6x + 8.5) = \pi$, then x is equal to

- (a) 2, 4
- (b) 1, 3
- (c) 3, 6
- (d) 4, 3

Answer: (a) 2, 4

Given, $6 * \sin^{-1} (x^2 - 6x + 8.5) = \pi$

$$\Rightarrow \sin^{-1} (x^2 - 6x + 8.5) = \pi/6$$

$$\Rightarrow x^2 - 6x + 8.5 = \sin \pi/6$$

$$\Rightarrow x^2 - 6x + 8.5 = 1/2$$

$$\Rightarrow 2x^2 - 12x + 17 = 1$$

$$\Rightarrow 2x^2 - 12x + 16 = 0$$

$$\Rightarrow x^2 - 6x + 8 = 0$$

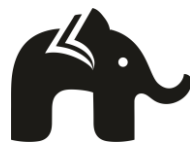
$$\Rightarrow (x - 2)(x - 4) = 0$$

$$\Rightarrow x = 2, 4$$

Question 22:

If $x + 1/x = 2$ then the principal value of $\sin^{-1} x$ is

- (a) $\pi/4$
- (b) $\pi/2$
- (c) π



(d) $3\pi/2$

Answer: (b) $\pi/2$

Given, $x + 1/x = 2$

$$\Rightarrow (x^2 + 1)/x = 2$$

$$\Rightarrow x^2 + 1 = 2x$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x - 1)^2 = 0$$

$$\Rightarrow x = 1$$

Now, $\sin^{-1} x = \sin^{-1} 1 = \pi/2$

So, the principal value of $\sin^{-1} x$ is $\pi/2$.

Question 23:

If $\cos^{-1} x > \sin^{-1} x$, then

(a) $1/\sqrt{2} < x \leq 1$

(b) $0 \leq x < 1/\sqrt{2}$

(c) $-1 \leq x < 1/\sqrt{2}$

(d) $x > 0$

Answer: (c) $-1 \leq x < 1/\sqrt{2}$

Given, $\cos^{-1} x > \sin^{-1} x$

$$\Rightarrow \pi/2 - \sin^{-1} x > \sin^{-1} x$$

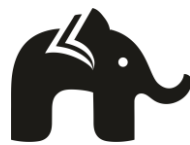
$$\Rightarrow \pi/2 > 2\sin^{-1} x$$

$$\Rightarrow \sin^{-1} x < \pi/4 \quad \dots\dots\dots 1$$

$$\text{Again, } -\pi/2 \leq \sin^{-1} x \leq \pi/2 \quad \dots\dots\dots 2$$

From equation 1 and 2, we get

$$-\pi/2 \leq \sin^{-1} x \leq \pi/4$$



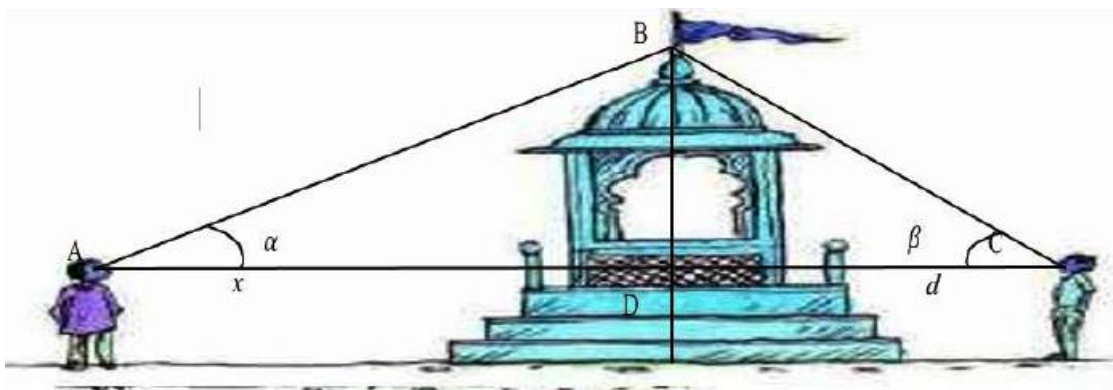
$$\Rightarrow \sin(-\pi/2) \leq x \leq \sin \pi/4$$

$$\Rightarrow -\sin \pi/2 \leq x \leq \sin \pi/4$$

$$\Rightarrow -1 \leq x \leq 1/\sqrt{2}$$

Case study based questions

Question 24:



Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters.

Based on the above information answer the following:

(i). $\angle CAB = \alpha =$

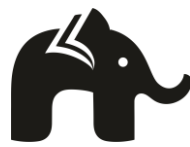
(a) $\sin^{-1}(2/\sqrt{3})$

(b) $\sin^{-1}(1/2)$

(c) $\sin^{-1}(2)$

(d) $\sin^{-1}(\sqrt{3}/2)$

(ii). $\angle CAB = \alpha =$



(a) $\cos^{-1}(1/5)$

(b) $\cos^{-1}(2/5)$

(c) $\cos^{-1}(\sqrt{3}/2)$

(d) $\cos^{-1}(4/5)$

(iii). $\angle BCA = \beta =$

(a) $\tan^{-1}(1/2)$

(b) $\tan^{-1}(2)$

(c) $\tan^{-1}(1/\sqrt{3})$

(d) $\tan^{-1}(\sqrt{3})$

(iv). $\angle ABC =$

(a) $\pi/4$

(b) $\pi/6$

(c) $\pi/2$

(d) $\pi/3$

(v). Domain and range of $\cos^{-1} x =$

(a) $(-1, 1), (0, \pi)$

(b) $[-1, 1], (0, \pi)$

(c) $[-1, 1], [0, \pi]$

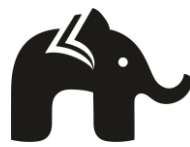
(d) $(-1, 1), [\pi/2, \pi/2]$

Answers:

(i). (b) $\sin^{-1}(1/2)$

In $\triangle ABD,$

$\tan \alpha = BD/AD$



$$\Rightarrow \tan \alpha = 30/30\sqrt{3}$$

$$\Rightarrow \tan \alpha = 1/\sqrt{3}$$

$$\Rightarrow \tan \alpha = \tan \pi/6$$

$$\Rightarrow \alpha = \pi/6$$

So, $\sin \alpha = \sin \pi/6$

$$\Rightarrow \sin \alpha = 1/2$$

$$\Rightarrow \alpha = \sin^{-1}(1/2)$$

(ii). (c) $\cos^{-1}(\sqrt{3}/2)$

From (i), $\alpha = \pi/6$

Now, $\cos \alpha = \cos \pi/6$

$$\Rightarrow \cos \alpha = \sqrt{3}/2$$

$$\Rightarrow \alpha = \cos^{-1}(\sqrt{3}/2)$$

(iii). (d) $\tan^{-1}(\sqrt{3})$

In $\triangle BCD$,

$$\tan \beta = BD/CD$$

$$\Rightarrow \tan \beta = 30/10\sqrt{3}$$

$$\Rightarrow \tan \beta = 3/\sqrt{3}$$

$$\Rightarrow \tan \beta = \sqrt{3}$$

$$\Rightarrow \beta = \tan^{-1}(\sqrt{3})$$

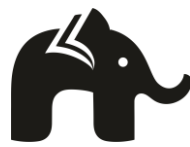
(iv). (c) $\pi/2$

We have $\alpha = \pi/6$, $\beta = \pi/3$

In $\triangle ABC$,

By Angle Sum Property

$$\alpha + \beta + \angle ABC = \pi$$



$$\pi/6 + \pi/3 + \angle ABC = \pi$$

$$\pi/2 + \angle ABC = \pi$$

$$\angle ABC = \pi - \pi/2$$

$$\angle ABC = \pi/2$$

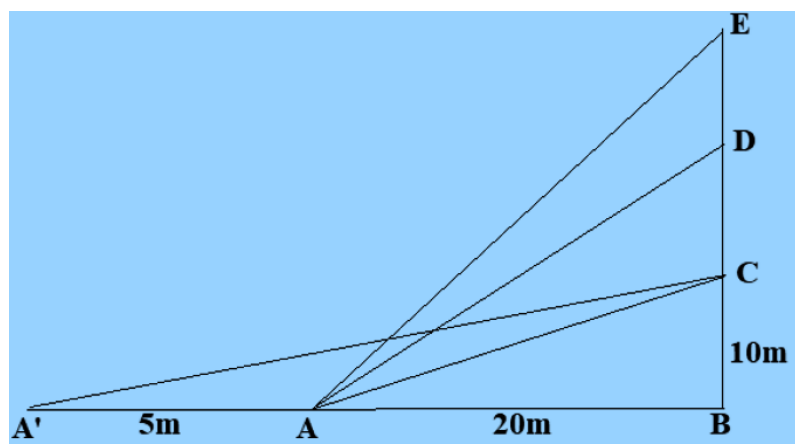
(v). (c) $[-1, 1], [0, \pi]$

Since, $\cos x$ is defined at $x = 0$ and $x = \pi$

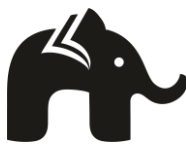
So, domain of $\cos^{-1} x$ includes -1 and 1 i.e. $[-1, 1]$

And range of $\cos^{-1} x$ also includes 0 and π i.e. $[0, \pi]$

Question 25:



The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C" The angle of elevation of "E" is triple the angle of



elevation of “C” for the same viewer. Look at the figure given and based on the above information answer the following:

(i). Measure of $\angle CAB =$

(a) $\tan^{-1}(2)$

(b) $\tan^{-1}(1/2)$

(c) $\tan^{-1}(1)$

(d) $\tan^{-1}(3)$

(ii). Measure of $\angle DAB =$

(a) $\tan^{-1}(3/4)$

(b) $\tan^{-1}(3)$

(c) $\tan^{-1}(4/3)$

(d) $\tan^{-1}(4)$

(iii). Measure of $\angle EAB =$

(a) $\tan^{-1}(11)$

(b) $\tan^{-1}(3)$

(c) $\tan^{-1}(2/11)$

(d) $\tan^{-1}(11/2)$

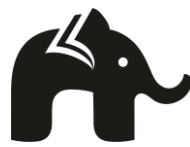
(iv). A' is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between $\angle CAB$ and $\angle CA'B$ is

(a) $\tan^{-1}(1/2)$

(b) $\tan^{-1}(1/8)$

(c) $\tan^{-1}(2/5)$

(d) $\tan^{-1}(11/21)$



(v). Domain and Range of $\tan^{-1} x$ is

(a) \mathbb{R}^+ , $(-\pi/2, \pi/2)$

(b) \mathbb{R}^- , $(-\pi/2, \pi/2)$

(c) \mathbb{R} , $(-\pi/2, \pi/2)$

(d) \mathbb{R} , $(0, \pi/2)$

Answer:

(i). (b) $\tan^{-1}(1/2)$

In ΔABC ,

$$\tan A = BC/AB$$

$$\Rightarrow \tan A = 10/20$$

$$\Rightarrow \tan A = 1/2$$

$$\Rightarrow A = \tan^{-1}(1/2)$$

$$\Rightarrow \angle CAB = \tan^{-1}(1/2)$$

(ii). (c) $\tan^{-1}(4/3)$

Given, $\angle DAB = 2 * \angle CAB$

$$= 2 * \tan^{-1}(1/2)$$

$$= \tan^{-1}\left[\frac{2 * 1/2}{1 - (1/2)^2}\right] \quad [\text{Apply } 2 \tan^{-1} x \text{ formula}]$$

$$= \tan^{-1}\left[\frac{1}{1 - 1/4}\right]$$

$$= \tan^{-1}\left[\frac{1}{3/4}\right]$$

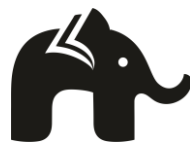
$$= \tan^{-1}(4/3)$$

(iii). (d) $\tan^{-1}(11/2)$

Given, $\angle EAB = 3 * \angle CAB$

$$= 3 * \tan^{-1}(1/2)$$

$$= \tan^{-1}\left[\frac{3 * (1/2) - (1/2)^3}{1 - 3 * (1/2)^2}\right]$$



$$\begin{aligned}
 & \text{[Apply 3 } \tan^{-1} x \text{ formula]} \\
 & = \tan^{-1}[(3/2 - 1/8)/(1 - 3/4)] \\
 & = \tan^{-1}[(11/8)/(1/4)] \\
 & = \tan^{-1}[(11/8) * (4/1)] \\
 & = \tan^{-1}(11/2)
 \end{aligned}$$

(iv). Answer does not matches

In $\Delta A'BC$,

$$\tan A' = BC/A'B$$

$$\Rightarrow \tan A' = 10/25$$

$$\Rightarrow \tan A' = 2/5$$

$$\Rightarrow A' = \tan^{-1}(2/5)$$

$$\Rightarrow \angle CA'B = \tan^{-1}(2/5)$$

$$\text{Now, } \angle CAB - \angle CA'B = \tan^{-1}(1/2) - \tan^{-1}(2/5)$$

$$= \tan^{-1}[(1/2 - 2/5)/(1 + 1/2 * 2/5)]$$

[Apply $\tan^{-1}a - \tan^{-1}b$ formula]

$$= \tan^{-1}[(1/10)/(6/5)]$$

$$= \tan^{-1}[(1/10) * (5/6)]$$

$$= \tan^{-1}(1/12)$$

(v). (c) $\mathbb{R}, (-\pi/2, \pi/2)$

Since $\tan x$ is not defined at $x = -\pi/2$ and $x = \pi/2$

So, Range of $\tan^{-1} x$ excludes $-\pi/2$ and $\pi/2$ i.e. $(-\pi/2, \pi/2)$

And domain of $\tan^{-1} x$ is all real number i.e. \mathbb{R}
