PRACTICE MCQs

CLASS 12 MATHS (TERM - I)

RELATION AND FUNCTIONS

BY

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learning simplified

Question 1:

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y): x is wife of y}, then R is ______ relation.

(a) Reflexive but not symmetric

(b) Symmetric but not transitive

(c) Transitive but not reflexive

(d) None of these

Answer: (d) None of these

Given, R = {(x, y): x is the wife of y}

Now, $(x, x) \notin R$

Since x cannot be the wife of herself.

So, R is not reflexive.

Now, let $(x, y) \in R$

=> x is the wife of y.

Clearly y is not the wife of x.

So, $(y, x) \notin R$

Indeed if x is the wife of y, then y is the husband of x.

So, R is not transitive.

Let $(x, y), (y, z) \in R$

=> x is the wife of y and y is the wife of z.

This case is not possible. Also, this does not imply that x is the wife of z.

=> $(x, z) \notin R$

So, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 2:

Let T be the set of all triangles in a plane with R a relation in T given by

R = {(T_1, T_2) : T_1 is congruent to T_2}. Then R is a/an _____ relation.

(a) reflexive

(b) symmetric

(c) transitive

(d) equivalence

Answer: (d) equivalence

R is reflexive, since every triangle is congruent to itself.

Again, $(T_1, T_2) \in R \Rightarrow T_1$ is congruent to T_2

$\Rightarrow T_2$ is congruent to T_1

$\Rightarrow (T_2, T_1) \in R$

Hence, R is symmetric.

Moreover,

$(T_1, T_2), (T_2, T_3) \in R \Rightarrow T_1$ is congruent to T_2 and T_2 is congruent to T_3

$\Rightarrow T_1$ is congruent to T_3

$\Rightarrow (T_1, T_3) \in R$

Hence, R is transitive.

Therefore, R is an equivalence relation.

Question 3:

The relation R on R defined as R = {(a, b): $a \leq b$} is

(a) reflexive, transitive and symmetric

(b) reflexive and symmetric but not transitive

(c) reflexive and transitive but not symmetric

(d) None of these

Answer: (c) reflexive and transitive but not symmetric

R = {(a, b): $a \leq b$}

Clearly (a, a) ∈ R [as a = a]

So, R is reflexive.

Now, (2, 4) ∈ R (as $2 < 4$)

But, (4, 2) ∉ R as 4 is greater than 2.

So, R is not symmetric.

Now, let (a, b), (b, c) ∈ R.

Then, $a \leq b$ and $b \leq c$

=> $a \leq c$

=> (a, c) ∈ R

So, R is transitive.

Hence R is reflexive and transitive but not symmetric.

Question 4:

A relation R in set A = {1, 2, 3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?

(a) (1, 1)

(b) (1, 2)

(c) (2, 2)

(d) (3, 3)

Answer: (b) (1, 2)

Given, relation R = {(1, 1), (1, 2), (2, 2), (3, 3)}

Here, (1, 2) is given.

So for symmetric, it should have (2, 1) also.

If we remove (1, 2), then the relation will be reflexive, symmetric and transitive.

Question 5:

The relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b} is a/an

(a) reflexive

(b) symmetric

(c) transitive

(d) equivalence

Answer: (d) equivalence

R is reflexive, as 2 divides (a – a) for all a ∈ Z.

Further, if (a, b) ∈ R, then 2 divides a – b.

Therefore, 2 divides b – a also.

Hence, (b, a) ∈ R, which shows that R is symmetric.

Similarly, if (a, b) ∈ R and (b, c) ∈ R, then a – b and b – c are divisible by 2.

Now, a – c = (a – b) + (b – c) is even. {Difference of two even number is even}

So, (a – c) is divisible by 2.

This shows that R is transitive.

Thus, R is an equivalence relation in Z.

Question 6:

The relation R in the set A = {x ∈ Z: 0 ≤ x ≤ 12}, given by

$$R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$$

is an equivalence relation. Then set of all elements related to 1 is

(a) {0, 1, 2, 5}

(b) A

(c) {1, 5, 9}

(d) {2, 5, 7}

Answer: (c) {1, 5, 9}

Given, R is an equivalence relation.

The set of elements related to 1 is {1, 5, 9} as

$|1 - 1| = 0$ is a multiple of 4.

$|5 - 1| = 4$ is a multiple of 4.

$|9 - 1| = 8$ is a multiple of 4.

Question 7:

Let L be the set of all lines in XY plane and R be the relation in L defined as

$R = \{(L_1, L_2): L_1$ is parallel to $L_2\}$. Then R is a/an

(a) reflexive

(b) symmetric

(c) transitive

(d) equivalence

Answer: (d) equivalence

Given, $R = \{(L_1, L_2): L_1$ is parallel to $L_2\}$
R is reflexive as any line L_1 is parallel to itself i.e., $(L_1, L_1) \in R$.

Now, let $(L_1, L_2) \in R$

$\Rightarrow L_1$ is parallel to L_2

$\Rightarrow L_2$ is parallel to L_1

⇒ $(L_2, L_1) \in R$

So, R is symmetric.

Now, let $(L_1, L_2), (L_2, L_3) \in R$

⇒ L_1 is parallel to L_2

Also, L_2 is parallel to L_3

⇒ L_1 is parallel to L_3

So, R is transitive.

Hence, R is an equivalence relation.

Question 8:

Let R be the relation in the set N given by R = {(a, b): a = b − 2, b > 6}. Choose the correct answer.

(a) (2, 4) ∈ R

(b) (3, 8) ∈ R

(c) (6, 8) ∈ R

(d) (8, 7) ∈ R

Answer: (c) (6, 8) ∈ R

Given, R = {(a, b): a = b − 2, b > 6}

Since b > 6, (2, 4) ∉ R

Also, as 3 ≠ 8 − 2

So, (3, 8) ∉ R

And, as 8 ≠ 7 − 2

So, (8, 7) ∉ R

Now, consider (6, 8).

We have 8 > 6 and also, 6 = 8 − 2

So, (6, 8) ∈ R

Question 9:

Let A be the set of all 50 students of Class X in a school. Let f : A →N be function defined by f (x) = roll number of the student x. Then f is

(a) one-one and onto

(b) one-one but not onto

(c) not one-one onto

(d) neither one-one nor onto

Answer: (b) one-one but not onto

No two different students of the class can have same roll number.

Therefore, f must be one-one.

We can assume without any loss of generality that roll numbers of students are from 1 to 50. This implies that 51 in N is not roll number of any student of the class, so that 51 cannot be image of any element of X under f.

Hence, f is not onto.

Question 10:

The function f is defined as f: Z → Z given by f(x) = x^2. Then f is

(a) injective but not surjective

(b) Not injective but surjective

(c) injective and surjective

(d) neither injective nor surjective

Answer: (d) neither injective nor surjective

f: Z → Z is given by $f(x) = x^2$

It is seen that f(-1) = f(1) = 1, but -1 ≠ 1

So, f is not injective.

Now, -2 ∈ Z. But, there does not exist any element x ∈ Z such that

f(x) = −2 or $x^2 = -2$

So, f is not surjective.

Hence, function f is neither injective nor surjective.

Question 11:

The function f: R⟶R defined as $(x) = x^3$ is:

(a) One-one but not onto

(b) Not one-one but onto

(c) Neither one-one nor onto

(d) One-one and onto

Answer: (d) One-one and onto

Given, f: R → R given by $f(x) = x^3$

It is seen that for x, y ∈ R,

f(x) = f(y)

$\Rightarrow x^3 = y^3$

⇒ x = y

So, f is one-one.

Again, let f(x) = y such that y ∈ R

=> $x^3 = y$

=> $x = y^{1/3}$

Here, y is a real number and for all values of y, we get value of x.

So, f is onto.

Hence, f is one-one and onto function.

Question 12:

The function f : N → N, given by f (1) = f (2) = 1 and f (x) = x – 1, for every x > 2, is

(a) onto and one-one

(b) not onto but not one-one

(c) onto but not one-one

(d) neither onto nor one-one

Answer: (c) onto but not one-one

f is not one-one, as f (1) = f (2) = 1

But f is onto, as given any y ∈ N, y ≠ 1,

We can choose x as y + 1 such that

f (y + 1) = y + 1 – 1 = y

Also for 1 ∈ N, we have f (1) = 1

So, f is onto but not one-one.

Question 13:

The Greatest Integer Function f: R → R given by f(x) = [x] where [x] denotes the greatest integer less than or equal to x, is

(a) one – one and onto

(b) one – one but not onto

(c) not one – one but onto

(d) neither one – one nor onto

Answer: (d) neither one – one nor onto

f: R → R is given by, f(x) = [x]

It is seen that f(1.2) = [1.2] = 1, f(1.9) = [1.9] = 1

So, f(1.2) = f(1.9), but 1.2 ≠ 1.9

Hence, f is not one – one.

Now, consider 0.7 ∈ R

It is known that f(x) = [x] is always an integer.

Thus, there does not exist any element x ∈ R such that f(x) = 0.7

So, f is not onto.

Hence, the greatest integer function is neither one – one nor onto.

Question 14:

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Then f is

(a) one-one

(b) onto

(c) not one – one

(d) not onto

Answer: (a) one-one

It is given that A = {1, 2, 3}, B = {4, 5, 6, 7}

f: A → B is defined as f = {(1, 4), (2, 5), (3, 6)}

So, f (1) = 4, f (2) = 5, f (3) = 6

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one – one.

Question 15:

Let f: R → R be defined as f(x) = x^4. Choose the correct answer.

(a) f is one-one onto

(b) f is many-one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto

Answer: (d) f is neither one-one nor onto

f: R → R is defined as $f(x) = x^4$

Let x, y ∈ R such that f(x) = f(y)

$\Rightarrow x^4 = y^4$

$\Rightarrow x = \pm y$

So, f(x) = f(y) does not imply that x = y

For example f(1) = f(-1) = 1

So, f is not one-one.

Consider an element 2 in co-domain R. It is clear that there does not exist any x in domain R such that f(x) = 2

So, f is not onto.

Hence, function f is neither one – one nor onto.

Question 16:

What type of relation is 'less than' in the set of real numbers?

(a) only symmetric

(b) only transitive

(c) only reflexive

(d) equivalence

Answer: (b) only transitive

If a ∈ R, a is not less than a.

=> The relation "less than" is not reflexive.

If a, b ∈ R, and a is less than b then b is not less than a.

=> The relation "less than" is not symmetric.

If a, b , c ∈ R, and a is less than b and b is less than c then a is less than c.

=>The relation "less than" is transitive.

Hence, the relation "less than" is also not an equivalence relation.

Question 17:

Given set A = {1, 2, 3} and a relation R = {(1, 2), (2, 1)}, the relation R will be

(a) reflexive if (1, 1) is added

(b) symmetric if (2, 3) is added

(c) transitive if (1, 1) is added

(d) symmetric if (3, 2) is added

Answer: (c) transitive if (1, 1) is added

Here, (1, 2) ∈ R, (2, 1) ∈ R

So, for transitive, (1, 1) should belong to R.

Question 18:

Given set A = {a, b, c). An identity relation in set A is

(a) R = {(a, b), (a, c)}

(b) R = {(a, a), (b, b), (c, c)}

(c) R = {(a, a), (b, b), (c, c), (a, c)}

(d) R= {(c, a), (b, a), (a, a)}

Answer: (b) R = {(a, a), (b, b), (c, c)}

A relation R is an identity relation in set A if for all a ∈ A, (a, a) ∈ R.

So, R = {(a, a), (b, b), (c, c)} is an identity relation.

Question 19:

Let S be the set of all real numbers. Then the relation R = {(a, b): $a^2 + b^2 = 1$} is

(a) symmetric and transitive

(b) symmetric and reflexive but not transitive

(c) symmetric but neither reflexive nor transitive

(d) symmetric, reflexive and transitive

Answer: (c) symmetric but neither reflexive nor transitive

(i) R is symmetric, since

$aRb \Rightarrow a^2 + b^2 = 1$

$\Rightarrow b^2 + a^2 = 1 \Rightarrow bRa$

(ii) R is not reflexive, since 1 is not related to 1, as

$1^2 + 1^2 = 1$ is not true.

(iii) Clearly, 1/2R√3/2 and √3/2R1/2

But 1/2 is not related to 1/2 since

$(1/2)^2 + (1/2)^2 = 1$ is not true.

So, R is not transitive.

Question 20:

Let S be the set of all real numbers and let R be a relation on S defined by

$aRb \Leftrightarrow |a| \le b$. Then, R is

(a) Reflexive but neither symmetric nor transitive

(b) Symmetric but neither reflexive nor transitive

(c) Transitive but neither reflexive nor symmetric

(d) Reflexive, symmetric and transitive

Answer: (c) Transitive but neither reflexive nor symmetric

Given, S = set of all real numbers.

And for a, b $\in$ S, aRb <=> $|a| \leq b$

Reflexive:

Take, $-1 \in S$, $|-1| = 1$

But $|-1| \leq 1$

=> $-1 \not R -1$

So, R is not reflexive.

Symmetric:

Take, $-2, 3 \in S$, such that $|-2| \leq 3$

But $|3| \leq -2$ is not true.

=> $3 \not R -2$

So, R is not symmetric.

Transitive:

Take, a, b, c $\in$ S, such that aRb, bRc i.e. $|a| \leq b$ and $|b| \leq c$

Since $|a| \leq b$

=> $|a| \leq |b| \leq c$

=> $|a| \leq c$

=> aRc

So, R is transitive.

Question 21:

The function f : R → R defined by f(x) = (x - 1)(x - 2)(x - 3) is

(a) one-one but not onto

(b) onto but not one-one

(c) both one-one and onto

(d) neither one-one and onto

Answer: (b) onto but not one-one

Given, $f(x) = (x - 1)(x - 2)(x - 3)$

One–one:

$\Rightarrow f(1) = (1 - 1)(1 - 2)(1 - 3) = 0$

$\Rightarrow f(2) = (2 - 1)(2 - 2)(2 - 3) = 0$

$\Rightarrow f(3) = (3 - 1)(3 - 1)(3 - 3) = 0$

$\Rightarrow f(1) = f(2) = f(3) = 0$

We can see, 1, 2, 3 has same image 0.

So, f is not one-one.

Onto:

Let y be an element in the co-domain R, such that

$y = f(x)$

$\Rightarrow y = (x - 1)(x - 2)(x - 3)$

Since, $y \in R$ and $x \in R$

So, f is onto.

Question 22:

Let $g(x) = x^2 - 4x - 5$, then

(a) g is one-one on R

(b) g is not one-one on R

(c) g is bijective on R

(d) None of these

Answer: (b) g is not one-one on R

Let $g(x_1) = g(x_2)$

$\Rightarrow x_1^2 - 4x_1 - 5 = x_2^2 - 4x_2 - 5$

$\Rightarrow x_1^2 - x_2^2 = 4(x_1 - x_2)$

=> $(x_1 - x_2)(x_1 + x_2 - 4) = 0$

=> $x_1 = x_2$ or $x_1 + x_2 = 4$

=> $x_1 = x_2$ or $x_1 = 4 - x_2$

Since, there are two values of x_1 for which $g(x_1) = g(x_2)$.

So, g(x) is not one-one $\forall x \in R$.

Question 23:

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Based on the given information, f is best defined as:

(a) Surjective function

(b) Injective function

(c) Bijective function

(d) function

Answer: (b) Injective function

Injective means one-one.

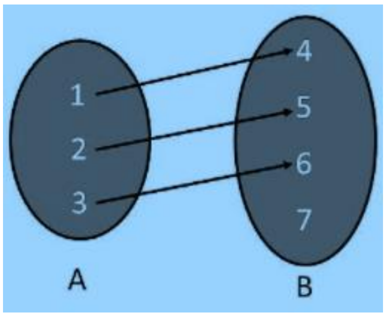
Since, all elements have unique image

So, R is injective

Surjective means onto.

Since, all elements do not have pre-image

So, R is not surjective.



Case study based questions

Question 24:

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever.

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

R = {$(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

(i). Two neighbors X and Y∈ I. X exercised his voting right while Y did not cast her vote in general election – 2019. Which of the following is true?

(a) (X, Y) ∈ R

(b) (Y, X) ∈ R

(c) (X, X) ∉ R

(d) (X, Y) ∉ R

(ii). Mr.'X' and his wife 'W'both exercised their voting right in general election -2019, Which of the following is true?

(a) both (X, W) and (W, X) ∈ R

(b) (X, W) ∈ R but (W, X) ∉ R

(c) both (X, W) and (W, X) ∉ R

(d) (W, X) ∈ R but (X, W) ∉ R

(iii). Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019, then which of the following is true?

(a) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

(b) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$

(c) $(F_1, F_2) \in R$, $(F_2, F_2) \in R$ but $(F_3, F_3) \notin R$

(d) $(F_1, F_2) \notin R$, $(F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

(iv). The above defined relation R is ________

(a) Symmetric and transitive but not reflexive

(b) Universal relation

(c) Equivalence relation

(d) Reflexive but not symmetric and transitive

(v). Mr. Shyam exercised his voting right in General Election – 2019, then Mr. Shyam is related to which of the following?

(a) All those eligible voters who cast their votes

(b) Family members of Mr. Shyam

(c) All citizens of India

(d) Eligible voters of India

Answers:

(i). (d) (X, Y) ∉ R

Given, R = {$(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

Since, X voted and Y did not vote.

So, we can write (X, Y) ∉ R.

(ii). (a) both (X, W) and (W, X) ∈ R

Given, R = {$(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

Since, X voted and W also voted.

So, we can write both (X, W) and (W, X) ∈ R.

(iii). (a) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

Given, R = {$(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

Since, all three friends voted.

So, $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

(iv). (c) Equivalence relation

Given, R = {$(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

Here, (V, V) ∈ R

So, R is reflexive.

If V_1 and V_2 both use their voting rights,

Then if $(V_1, V_2) \in R$ then $(V_2, V_1) \in R$

Hence, R is symmetric.

If $(V_1, V_2) \in R$ then $(V_2, V_3) \in R$

Then $(V_1, V_3) \in R$

Hence, R is transitive.

Since R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

(v). (a) All those eligible voters who cast their votes

Given, R = $\{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

So, Mr. Shyam will be related to all eligible voters who casted their votes.

Question 25:

Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

Answer the following questions using the above information.

(i). Let $f: R \to R$ be defined by $(x) = x^2$ is__________

(a) Neither Surjective nor Injective

(b) Surjective

(c) Injective

(d) Bijective

(ii). Let $f: N \rightarrow N$ be defined by $(x) = x^2$ is ________

(a) Surjective but not Injective

(b) Surjective

(c) Injective

(d) Bijective

(iii). Let f: {1, 2, 3, ….} → {1, 4, 9, ….} be defined by $f(x) = x^2$ is _________

(a) Bijective

(b) Surjective but not Injective

(c) Injective but Surjective

(d) Neither Surjective nor Injective

(iv). Let : $N \rightarrow R$ be defined by $f(x) = x^2$. Range of the function among the following is _________

(a) {1, 4, 9, 16, …}

(b) {1, 4, 8, 9, 10, …}

(c) {1, 4, 9, 15, 16, …}

(d) {1, 4, 8, 16, …}

(v). The function f: Z → Z defined by $(x) = x^2$ is__________

(a) Neither Injective nor Surjective

(b) Injective

(c) Surjective

(d) Bijective

Answer:

(i). (a) Neither Surjective nor Injective

Given, $f(x) = x^2$, where f : R - > R

For one-one:

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

Now, $f(x_1) = f(x_2)$

=> $(x_1)^2 = (x_2)^2$

=> $x_1 = x_2$ or $x_1 = -x_2$

Since x_1 does not have unique image

So, it is not one-one.

For onto:

Let f(x) = y such that $y \in R$

=> $x^2 = y$

=> $x = \pm\sqrt{y}$

Since, y is a real number, so it can be negative also and root of a negative number is not real.

So, x is not real.

=> f is not onto.

Thus, f(x) is neither Surjective nor injective.

(ii). (c) Injective

Given, $f(x) = x^2$, where f : N - > N

For one-one:

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

Now, $f(x_1) = f(x_2)$

=> $(x_1)^2 = (x_2)^2$

=> $x_1 = x_2$ or $x_1 = -x_2$

Since x_1 cannot be negative since x_1 and x_2 are natural numbers.

So, only option is

When $f(x_1) = f(x_2)$, then $x_1 = x_2$

Hence, f is one-one

For onto:

Let $f(x) = y$ such that $y \in \mathbb{N}$

=> $x^2 = y$

=> $x = \pm\sqrt{y}$

Since, x is a natural, it will be positive.

So, $x = \sqrt{y}$

Also, since y is a natural number, let's take y = 2

Now, $x = \sqrt{2}$ which not possible since x is a natural number.

So, f is not onto.

(iii). (a) Bijective

Given, $f(x) = x^2$, where $f : \{1, 2, 3, \ldots\} \to \{1, 4, 9, \ldots\}$

For one-one:

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

Now, $f(x_1) = f(x_2)$

=> $(x_1)^2 = (x_2)^2$

=> $x_1 = x_2$ or $x_1 = -x_2$

Since $x \in \{1, 2, 3, ..\}$, x cannot be negative

So, only option is

When $f(x_1) = f(x_2)$, then $x_1 = x_2$

Hence, f is one-one

For onto:

Let f(x) = y such that y ∈ {1, 4, 9, ...}

=> $x^2 = y$

=> x = ±√y

Since, x ∈ {1, 2, 3, ..}

So, x = √y

Also, since y ∈ {1, 4, 9, ..},

For all values of y, we will get a value of x, where x ∈ {1, 2, 3, ..}

So, f is onto.

Since f is one-one and onto,

Hence, f is bijective.

(iv). (a) {1, 4, 9, 16, ...}

For f(x) = x^2, where f : N -> R

Range will be = {1^2, 2^2, 3^2, 4^2, ...} = {1, 4, 9, 16, ...}

(v). (a) Neither Injective nor Surjective

Given, f(x) = x^2, where f : Z - > Z

For one-one:

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

Now, $f(x_1) = f(x_2)$

=> $(x_1)^2 = (x_2)^2$

=> $x_1 = x_2$ or $x_1 = -x_2$

Since x_1 does not have unique image,

Hence, f is one-one.

For onto:

Let f(x) = y such that y ∈ $\mathbb{Z}$

=> $x^2 = y$

=> $x = \pm\sqrt{y}$

Since y is an integer, let’s take y = 5

Now, $x = \pm\sqrt{5}$ which not possible since √5 is not an integer.

So, f is not onto.

Hence, f(x) is neither Surjective nor injective.

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